LABOR MARKET FRICTIONS, JOB INSTABILITY, AND THE FLEXIBILITY OF THE EMPLOYMENT RELATIONSHIP[*]

Niko Matouschek
Northwestern University

Paolo Ramezzana
Frontier Economics

Frédéric Robert-Nicoud
London School of Economics

Abstract

We endogenize separation in a search model of the labor market and allow for bargaining over the continuation of employment relationships following productivity shocks to take place under asymmetric information. In such a setting separation may occur even if continuation of the employment relationship is privately efficient for workers and .rms. We show that reductions in the cost of separation, owing for example to a reduction in .ring taxes, lead to an increase in job instability and, when separation costs are initially high, may be welfare decreasing for workers and firms. We furthermore show that, in response to an exogenous reduction in .ring taxes, workers and firms may switch from rigid to flexible employment contracts, which further amplifies the increase in job instability caused by policy reform.

---

1 Introduction

Bargaining between firms and workers often takes place in the presence of private information. Suppose, for instance, that a firm adopts a new production technology and that this technology is only effective if workers invest in learning how to use it. It is likely that the firm is not perfectly informed about the workers' costs of making such investments and that the workers are imperfectly informed about the firm's benefits of adopting the new technology. Negotiations over the continuation of the employment relationships will then be hampered by the presence of private information. The purpose of this paper is to analyze the functioning of decentralized labor markets under such circumstances. To this end we endogenize separation in a search model of the labor market and allow for negotiations over separations to take place under private information.

In most existing search models, bargaining between firms and workers takes place under perfect information (see e.g. Pissarides 2000). An important implication of this assumption is that bargaining is efficient, that is, firms and workers instantaneously agree to form, or continue, employment relationships if there are gains from trade and they instantaneously agree to separate if there are none. In contrast, the parties may fail to realize all gains from trade if bargaining takes place under asymmetric information (see Myerson and Satterthwaite 1983, Hall and Lazear 1984, and Hall 1995).[1] In this paper we show that the inefficiencies that arise when firms and workers bargain over separations in the presence of private information have important implications for our understanding of labor markets and the contractual relationships between firms and workers. In particular, our paper offers three main insights.

---

[1] There is a large experimental literature that supports the view that agents often fail to reach efficient agreements (see Kagel and Roth 1997).

First, we show that in a labor market in which workers and firms bargain under asymmetric information, a reduction in the cost of separation leads to an increase in job instability, i.e. in the probability that workers and firms separate following productivity shocks. The reason for this is that a reduction in separation costs increases the joint value of the workers and the firms' outside options and thus induces them to bargain more aggressively. This, in turn, makes it more likely that firms and workers will separate following productivity shocks and thus makes employment relationships more unstable. In this regard it is important to note that, contrary to what would happen in a model in which workers and firms bargain under perfect and symmetric information, in our model workers and firms may sometimes separate even when it would be efficient for them not to do so, i.e. even when their joint expected utility from continuing the employment relationship is greater than their joint expected utility from separating and returning to the labor market.

Second, we show that, because separations can be privately inefficient, a reduction in the cost of separation can make some groups in society – and possibly society as a whole – worse off by increasing the probability of separation. To see this, consider the effect of a reduction in the firing tax that has to be paid whenever a firm and a worker separate. On the one hand, such a reduction has a direct positive effect on the worker and the firm's expected welfare, because it lowers separation costs when the productivity shock is so large that separation is actually privately efficient. On the other hand, however, precisely because it becomes less costly to separate, the reduction in the firing tax induces the workers and firms to bargain more aggressively and increases the probability of privately inefficient separation. This negative effect can dominate the direct benefit of a reduction in firing taxes, with the possible consequence that firms and workers can be actually made worse off by a reduction in these taxes. Our analysis

allows us to derive precise, and potentially testable, predictions about the conditions under which different groups in society are made worse off by a labor market reform. As such our analysis contributes to the literature that seeks to explain why reforms that reduce labor market frictions have often faced substantial political opposition in spite of their strong support by institutions such as the OECD and the IMF (see, for instance, Saint-Paul 2000 and Pissarides 2001).

Third, our analysis contributes to our understanding of contractual arrangements between firms and workers and of the effects of changes in the labor market environment on these arrangements. Firms and workers often use employment contracts that make it *more* costly for them to separate in the future. For instance, since the mid-1980s firms in Europe have been able to choose between hiring workers on 'rigid' permanent contracts for which separation costs are significant or on 'flexible' fixed-term contracts for which they are negligible.[2] We provide a novel explanation for why firms and workers may agree to adopt rigid employment contracts that increase future separation costs. In particular, we show that firms and workers that anticipate future bargaining inefficiencies can use these contracts as a means of committing to less aggressive bargaining behavior and thus 'stabilize' their employment relationship. Our model shows that whether or not firms and workers want to use such contracts for this purpose depends crucially on the labor market environment and the workers' productivity. In particular, it shows that a sufficiently large reduction in exogenously imposed separation costs, such as a reduction in firing taxes, leads to a one-off switch from rigid to flexible employment contracts. This endogenous contractual response to exogenous labor policy changes induces firms and workers to bargain even more aggressively and thus further destabilizes employment

---

[2]The 'difficulty of dismissal' is one of three kinds of employment protection listed by the OECD that increases firing costs but does not involve any transfer from the employer to the employee (see also Footnote 9 below).

relationships.

We discuss the related literature in the next section after which we present the structure of the model. In Section 4 we solve the model, taking as given the contractually specified separation costs and discuss the effects of a reduction in firing taxes. We then endogenize the contractually specified separation costs in Section 5, check for the robustness of our results in Section 6 and finally conclude in Section 7. All proofs are in the Appendix.

2   Related Literature

Our paper is related to a number of contributions that have identified different mechanisms through which a reduction in market frictions can be welfare reducing. It has been shown, for instance, that in a repeated games setting a reduction in market frictions can increase the reneging payoff of agents and thereby hamper cooperation and potentially reduce welfare (Kranton 1996, McLaren and Newman 2002). It has also been shown that a reduction in market frictions can have adverse effects on the investment incentives of agents and reduce welfare through this channel (Ramey and Watson 2001). There are also a number of papers that have identified reasons why market frictions in the form of employment protections can be welfare enhancing. For instance, it has been shown that when firms rely on efficiency wages to motivate workers, severance payments can mitigate the tendency of the firms to fire workers 'too often' and, through this channel, increase welfare (Saint Paul 1995, 1996, Fella 2000). Pissarides (2001) shows that different kinds of employment protection can serve as insurance devices when workers are risk averse and Alvarez and Veracierto (2001) show that severance payments can

increase welfare by providing insurance and reducing firings.[3] Our paper is related to these contributions since we show that one consequence of inefficient bargaining between firms and workers is that an increase in separation costs can be welfare enhancing.

While a number of papers have shown that severance payments can be welfare enhancing, less work has been done one why such payments should be government imposed rather than left to private contracts. One paper that does address this point directly is Schmitz (2004) who provides a model of an employment relationship with asymmetric information and shows that even if firms and workers can write any contract they may be better off when the government imposes employment protections. While the focus of our paper is different, our analysis is related to this paper in so far as we too allow for the possibility of private contracting over employment protections.[4]

There is a large literature that analyzes bargaining under asymmetric information (for an overview see Ausubel, Cramton and Deneckere 2002). In a seminal contribution, Myerson and Satterthwaite (1983) analyze bilateral bargaining between two agents with private and independent valuations. They show that if trade is not always efficient and the agents can opt out of the bargaining game after having learned their valuations, bargaining is necessarily inefficient, that is, the agents will sometimes fail to realize strictly positive gains from trade. They also characterize the second best trading mechanism.[5] Most of the literature on bilateral bargaining under private information assumes that agents are risk neutral and we follow the literature in this respect. One recent exception is Copic and Ponsati (2004) who consider a similar setting as Myerson and Satterth-

---

[3]A related literature studies the introduction of fixed-term contracts. See Güell (2005) and Alonso-Borrego, Fernandez-Villaverde and Galdon-Sanchez (2004) and the references therein.

[4]Burguet, Caminal and Matutes (2002) consider private contracting over employment protections (buy-out fees and severance payments) when the incumbent employer has more information about an employee than the labor market has.

[5]Williams (1999) extends the results in Myerson and Satterthwaite (1983) to a multilateral setting.

waite (1983) and allow for risk aversion by the traders. They characterize the optimal mechanism under ex post individual rationality and ex post incentive compatibility. It would be interesting to allow for risk aversion by the workers in setting such as ours but we do not address this issue in this paper.

Several recent papers study the consequences of asymmetric information in models of competitive search (Shimer and Wright 2004, Faig and Jerez 2005, Moen and Rosen 2005). In this literature the paper most closely related to ours is Guerrieri (2005), who analyzes a competitive search model in which workers are privately informed about their disutility of labor and shows that, in a dynamic version of the model, the equilibrium can be constrained inefficient. Her paper and ours are related since in both the workers' endogenous outside options determine their participation constraints and, through this channel, the cost of information revelation. However, whilst she focuses on the constrained efficiency of a competitive search equilibrium, we focus on the consequences of inefficient bargaining for job instability and the types of labor contracts adopted in equilibrium.

Finally, our paper is related to Matouschek and Ramezzana (2007) who develop a model of a matching market in which buyers and sellers can adopt exclusive contracts to reduce future bargaining inefficiencies. Although their paper also studies bargaining inefficiencies in a matching market, it focuses on the search externalities imposed by a buyer-seller pair on other buyers and sellers when they adopt exclusive contracts rather than on the effects of policy reform and contractual choices on the instability of trade relationships over time like the present paper.

3   The Model

We consider a dynamic market in which time is discrete and runs indefinitely. All agents are risk-neutral, liquidity unconstrained, and discount the future at rate r.

There are two types of agents: 'workers' and 'firms.' At the beginning of every period an exogenous and equal number n of new workers and of new firms enter the market. Every worker enters the market as unemployed and searches for a job, while every firm enters the market with an open vacancy and searches for an employee. In any period in which a worker is unemployed he receives utility b and in any period in which a firm has an open vacancy it incurs costs c. Unemployed workers and firms with open vacancies can decide to leave the market permanently at any point in time and obtain utility equal to zero for ever. If they instead decide to stay in the market, they are brought together by a random matching process, that we describe further below. If, after having met, a firm and a worker decide to start an employment relationship then, in every subsequent period in which they are still together, the firm produces one unit of a good worth p and the worker experiences a disutility that, without loss of generality, is set equal to zero. The worker's disutility and the firm's marginal product p are common knowledge and the marginal product is sufficiently large to ensure that all employment relationships are initially formed. The firm and the worker bargain over the wage at the beginning of their employment relationship and renegotiate it in every period until they separate. We assume that when wages are negotiated the firm and the worker each make a take-it-or-leave-it offer with probability $1/2$. If the offer is accepted, production takes place, wages are paid and time moves on to the next period. If the offer is rejected, the worker becomes unemployed and receives b, the firm reopens the vacancy and incurs costs c and time moves on to the next period. Employment relationships are

subject to random shocks that lead the worker and the firm to negotiate compensation under private information, as described further below. Furthermore, at the end of every period the worker dies or the vacancy is destroyed with exogenous probability d. For simplicity, we assume that if the worker dies the vacancy that he occupied is destroyed and if the vacancy is destroyed the worker who occupied it must also leave the market. The effective discount rate is given by $\delta \equiv (1-d)/(1+r)$.

Vacancies and unemployed workers are brought together by a random matching process. In particular, at the beginning of every period the total number of contacts is given by the matching function $M = am(u,v) \leq \min\{u,v\}$, where u and v denote the number of unemployed workers and of open vacancies, respectively. We assume that this function is increasing, continuous, and homogeneous of degree one in both arguments; we also normalize $m(1,1)$ to unity. Since unemployed workers and open vacancies enter the market in equal numbers and they only leave the market in pairs, we have that $u = v$ and thus that $M/u = M/v = a$. In other words, unemployed workers and firms with open vacancies find a counterpart with the same exogenously given probability a. The parameter a, therefore, captures the efficiency of the matching process.

The focus of our analysis is on job destruction, which we describe next. At the beginning of every period existing employment relationships are hit by a shock with probability λ. To avoid having to discuss uninteresting cases, we assume that the shock does not hit very often, in the sense that $\lambda < (1-\delta)/(2\delta)$. When an employment relationship is hit by a shock, the firm and the worker need to decide between either making a costly adjustment and continuing their relationship or separating. The type of adjustment that we have in mind is, for instance, a fundamental change in the production technique or a reorganization of the firm. If the adjustment is made, the worker experiences a one-off disutility γ and the firm realizes a, potentially negative, one-off

benefit ρ. After γ and ρ have been incurred, the firm and the worker can continue their employment relationship as described above, i.e. they can produce one unit of a good worth p after which time moves on to the next period. If the adjustment is not made, the worker-firm pair becomes unproductive and separates. The worker's disutility, γ, is drawn from a distribution with cumulative function $G(\gamma)$ and density function $g(\gamma)$ on some compact support Γ. We denote the inverse of the hazard rate by $H(\gamma) \equiv G(\gamma)/g(\gamma)$ and assume that it is monotonically increasing. Analogously, the firm's realization of ρ is drawn from a distribution with cumulative function $F(\rho)$ and density function $f(\rho)$ on some compact support P. We denote the inverse of the hazard rate of this distribution by $K(\rho) \equiv [1 - F(\rho)]/f(\rho)$ and assume that it is monotonically decreasing.[6] Our key assumption is that, although the distributions $F(\rho)$ and $G(\gamma)$ are common knowledge, the realizations ρ and γ are privately observed by the firm and the worker, respectively. Bargaining over the amount that the firm should pay the worker for the adjustment is therefore hindered by the presence of private information. To be consistent with the wage bargaining process in the absence of shocks, we assume that when a shock occurs the firm and the worker can each make a take-it-or-leave-it offer with probability 1/2. If this offer is accepted, the worker and the firm continue their employment relationship and if it is rejected they separate and return to the search pool. We also assume that the γ's and the ρ's are uncorrelated between them and over time. Finally, we assume that Γ and P are such that for some realizations of γ and ρ it is efficient for the firm and the worker to make the adjustment while for other realizations it is not.[7] The timing of the bargaining game is summarized in Figure 1.

Whenever a firm and worker who are currently in an employment relationship sepa-

---

[6]The monotone hazard rate conditions imposed on $H(\cdot)$ and $K(\cdot)$ are satisfied by many common distributions.

[7]We are ruling out the uninteresting cases in which it is always efficient to make the adjustment and in which it is never efficient to do so.

rate, the firm must bear two types of costs: a firing tax ϕ, which is fixed exogenously by the government, and, possibly, a self-imposed cost $R \in [0, \overline{R}]$ to which the firm and the worker commit at the beginning of their employment relationship.[8] The upper limit $\overline{R}$ is exogenously given. Note that R is not a transfer from the firm to the worker, but it is a pure cost that detracts from their joint surplus, e.g. costly legal or administrative procedures, an amount paid to an escrow fund, a net loss on retirement income, or investments that are partly specific to the firm worker pair and that cannot be fully recovered by either party.[9] As we shall discuss in Section 5, in a world with public information agents would always set $R = 0$ but this is no longer the case under private information.

Having outlined the structure of the model, it is worth briefly discussing the reasons for and the implications of two of our simplifying assumptions. First, we rule out free entry by firms and focus on a simple model with an exogenous inflow of firms which is equal to the inflow of workers. This has the advantage of simplifying the analysis, since the market tightness ratio of vacancies to unemployed workers is always unity. It can be shown, however, that our main results continue to hold in a version of the model with free entry, as discussed in Section 4.5. Second, for most of the paper we assume that the players use a given, simple bargaining game in their negotiations following a shock. A possible concern with this approach is, however, that our results may depend on this particular bargaining game, especially in light of the fact that this is not the

---

[8]The results that we derive in this paper would not be changed if the extra costs R are incured not only by the firm but also by the worker.

[9]Pissarides (2001, page 136) discusses the five kinds of employment protection listed by the OECD and argues that three of them (administrative procedures, difficulty of dismissal and additional measures for collective dismissals) "appear to be mainly ways of making it difficult for the employer to dismiss a worker without any apparent immediate financial gain to the employee." Saint Paul (1995, page 49) argues that firms can use a technology with costly training and specific investment in human capital to make it more costly for themselves to fire workers in the future. Fella (2000) also allows for the possibility that the severance payment that a worker receives is less than the amount paid by the firm.

most efficient bargaining game that the parties could adopt.[10]   To address this concern,
in Section 6 we assume that firms and workers adopt the most efficient bargaining game
and show that our qualitative results are unaffected by the adoption of this more general
framework.

4   Equilibrium without Self-imposed Rigidities

In this section we study the equilibrium of our model in the case in which the firm and
the worker cannot contract over the self-imposed rigidities.   Thus, for the time being,
R is taken as an exogenous parameter.   We solve the model in Sections 4.1-4.3 and
analyze the effect of a fall in the firing tax in Section 4.4.

4.1   Job Creation and Wage Setting

A firm and a worker who are matched first negotiate an initial wage and then, in case
they agree on a wage and form an employment relationship, renegotiate the wage in
every subsequent period in which they are still together.   In this section we consider
wage bargaining between the firm and the worker at the initial hiring stage and in all
subsequent periods in which there is no productivity shock.   Note that, in contrast
to the bargaining over the adjustment bonus considered in the next subsection, in this
section wage bargaining always takes place under public information.

Consider then a firm and a worker who have just been matched.   If they form an
employment relationship, they can generate a match value Z.   If, instead, they return
to the search pool, the worker and the firm realize U and V respectively, where U is the
worker's expected utility when unemployed and V is the expected value of a firm when

---

[10]The simple bargaining game used in most of the paper is not the most efficient one in the sense that
if firms and workers were able to contract over the bargaining game when they form their employment
relationship, they would choose a different one.

it is searching for a worker. Wages are determined by a bargaining game in which the firm and the worker each make a take-it-or-leave-it wage offer with probability $1/2$. In this game it is optimal to offer a wage that makes the other party indifferent between accepting and rejecting. Thus, whenever the firm makes an offer, it offers a wage such that the worker's lifetime expected utility from being employed by the firm, W, is equal to U, his expected utility when unemployed. Similarly, when the worker makes the offer, he demands a wage such that the firm's expected value from employing the worker, J, is equal to the firm's expected value when searching for a worker V. It then follows that when a firm and a worker are initially matched, the worker's lifetime expected utility is

$$W_0 = U + \frac{1}{2}[Z - S_0] \tag{1}$$

and the expected value of the firm is

$$J_0 = V + \frac{1}{2}[Z - S_0], \tag{2}$$

where

$$S_0 \equiv U + V \tag{3}$$

is the joint value of search if the firm and the worker do not form an employment relationship.[11]

Consider next a firm and a worker who are renegotiating the wage in some period after they formed their employment relationship. When it is the firm's turn, it still offers a wage such that $W = U$. When it is the worker's turn, however, he will now offer a wage such that $J = V - \phi - R$, where ϕ is the firing tax and R the self-imposed rigidities. This is the case since, once an employment relationship has been formed, the firm can only return to the search pool and realize J after incurring separation costs

---

[11] Since the dynamic structure of our model is fairly straightforward, we omit time subscripts throughout the paper in order to simplify notation.

equal to $(\phi + R)$. In every period in which the worker-firm pair has not been hit by a productivity shock, the worker's lifetime expected utility is then given by

$$W_1 = U + \frac{1}{2}[Z - S] \tag{4}$$

and the expected value of the firm is given by

$$J_1 = V - \phi - R + \frac{1}{2}[Z - S], \tag{5}$$

where $S \equiv U + V - \phi - R$ is the joint value of search if the firm and the worker terminate their employment relationship. Note that when the separation costs are strictly positive, that is when $\phi + R > 0$, the value of the firm is higher at the beginning than during an employment relationship, i.e. $J_0 > J_1$, while the expected utility of the worker is higher during an employment relationship than at its beginning, i.e. $W_1 > W_0$. This is the case since, with strictly positive separation costs, the firm is in a stronger position when it bargains with a new potential worker than when it bargains with an existing employee.

Note also that in contrast to its distribution, the size of the match value does not depend on whether the worker and the firm are at the start or at some later stage of their employment relationship. In particular, using (1) to (5), we have

$$W_0 + J_0 = W_1 + J_1 = Z. \tag{6}$$

The match value Z is given by

$$Z = p + \delta \left[\lambda Z^e + (1 - \lambda)Z \right],$$

where λ is the probability that a shock hits and Z^e is the expected match value when a shock hits. Rearranging then gives

$$Z = \frac{1}{1 - \delta(1 - \lambda)}(p + \lambda \delta Z^e). \tag{7}$$

13

4.2 Job Destruction

Consider now the implications of the model outlined in Section 3 for job destruction. When the shock occurs, the firm and the worker negotiate over the adjustment bonus β and these negotiations are hindered by the presence of private information. In particular, with probability $1/2$ the firm makes a take-it-or-leave-it offer without observing the worker's disutility γ and with probability $1/2$ the worker makes a take-it-or-leave-it offer without observing the firm's realization of the adjustment benefit ρ.

Suppose first that the firm makes the offer. The worker accepts an offer β if and only if the value of continuing with the current employer is greater than or equal to the value of search U, i.e. if and only if $\beta - \gamma + W_1 \geq U$. Rearranging one obtains that the worker continues in the job if and only if

$$\gamma \leq \widetilde{\gamma} \equiv \beta + W_1 - U, \tag{8}$$

that is, if the adjustment bonus β and the net continuation payoff $W_1 - U$ are sufficiently large relative to the adjustment cost γ.

The firm chooses the bonus β in order to maximize its expected value from the time of the shock onward, given by $G(\widetilde{\gamma})\,(\rho - \beta + J_1) + [1 - G(\widetilde{\gamma})](V - \phi - R)$. Since (8) establishes a one-to-one relationship between the bonus β offered by the firm and the marginal worker $\widetilde{\gamma}$ who accepts it, in order to solve the firm's expected profit maximization problem we can think of the firm choosing the marginal worker $\widetilde{\gamma}$. Therefore using (8) to substitute for β and then (6) to substitute Z for $(W_1 + J_1)$ in the expression for the expected value of the firm given above, we have that a firm with productivity ρ solves

$$\max_{\widetilde{\gamma}}\ G(\widetilde{\gamma})\,(\rho - \widetilde{\gamma} + Z - U) + [1 - G(\widetilde{\gamma})]\,(V - \phi - R). \tag{9}$$

The first order condition of this problem is

$$\widetilde{\gamma} + H(\widetilde{\gamma}) = \rho + Z - S, \tag{10}$$

for all ρ and where $H(\gamma)$ is the inverse of the hazard rate. Equation (10) implies that, although we do not make this explicit in the notation, $\widetilde{\gamma}$ is a function of ρ.

Suppose now that the worker makes the offer and demands a bonus β in order to agree to continue in its current job. The firm accepts this offer if and only if the value of continuing with the current worker is greater than or equal to the value of search, i.e. if and only if $\rho - \beta + J_1 \geq V - \phi - R$. Rearranging this expression one obtains that the firm continues to employ the worker if and only if

$$\rho \geq \widetilde{\rho} \equiv V - \phi - R + \beta - J_1. \tag{11}$$

The worker chooses the bonus β in order to maximize his expected utility from the time of the shock onward, given by $(1 - F(\widetilde{\rho})) (\beta - \gamma + W_1) + F(\widetilde{\rho})U$. Using (11) to substitute for β and then (6) to substitute Z for $(W_1 + J_1)$ in the expression above, the worker's offer solves

$$\max_{\widetilde{\rho}} (1 - F(\widetilde{\rho})) (\widetilde{\rho} - \gamma + Z - (V - \phi - R)) + F(\widetilde{\rho})U. \tag{12}$$

The first order condition of this problem is

$$\widetilde{\rho} - K(\widetilde{\rho}) = \gamma - Z + S, \tag{13}$$

for all γ; where $K(\rho)$ is the inverse of the hazard rate. Note that, although we do not make this explicit in the notation, $\widetilde{\rho}$ is a function of γ.

Note also that it would be ex post efficient for the worker and the firm to continue their employment relationship whenever

$$\rho - \gamma + Z \geq S. \tag{14}$$

15

The presence of $H(\widetilde{\gamma}) > 0$ and $K(\widetilde{\rho}) > 0$ in the first order conditions (10) and (13) implies that the firm and the worker sometimes separate when it would be efficient for them to continue their employment relationship, i.e. they separate for some ρ and γ that satisfy (14). This bargaining inefficiency is at the heart of our analysis.

For the analysis that follows it is useful to derive an expression for the expected match value when the firm-worker pair is hit by a shock Z^e. Every time a firm-worker pair is hit by a shock they jointly realize $\rho - \gamma + Z$ if they continue their relationship and they jointly realize S if they separate. Thus, Z^e is given by

$$
\begin{aligned}
Z^e \;=\; & S + \frac{1}{2}\int_{\rho \in \mathrm{P}}\int_{\gamma \leq \widetilde{\gamma}}(\rho - \gamma + Z - S)\,\mathrm{d}G(\gamma)\mathrm{d}F(\rho) \\
& + \frac{1}{2}\int_{\gamma \in \Gamma}\int_{\rho \geq \widetilde{\rho}}(\rho - \gamma + Z - S)\,\mathrm{d}F(\rho)\mathrm{d}G(\gamma).
\end{aligned}
\tag{15}
$$

4.3 Steady-state Market Equilibrium

A steady-state equilibrium of the model without self-imposed rigidities is a tuple $\langle \widetilde{\gamma}, \widetilde{\rho}, U, V\rangle$ that satisfies the firms and workers utility maximization problems in (9) and (12), respectively, and equations (1) - (7) and (15).[12]

We start by solving for U and V. Since an unemployed worker can always drop out of the labor market and obtain a utility equal to zero, his expected utility is the maximum between zero and the value of search, which is given by $[b + \delta[aW_0 + (1-a)U]$, that is

$$
U = \max\{b + \delta\left[aW_0 + (1-a)U\right], 0\}.
\tag{16}
$$

where a is the probability with which firms and workers meet a counterpart and W_0 is

---

given in equation (1). Similarly, the value of a vacancy, V, must satisfy

$$V = \max\left\{-c + \delta\left[aJ_0 - (1-a)V\right], 0\right\}, \tag{17}$$

where J_0 is given in equation (2).

Note that if one side of the market (e.g. workers) does not decide to participate in the labor market, then it is an optimal response for the other side (e.g. firms) to also stay out of the market. In what follows we disregard this type of trivial equilibria with no trade – which always exist in any two-sided search model – and focus on equilibria in which both parties participate in the market. In particular, we assume that the parameters of the model are such that there always exist equilibria in which $U > 0$ and $V > 0$.[13]

Adding (16) and (17) we have that the joint value of search, S_0, for a firm-worker pair who decide not to start an employment relationship satisfies

$$(1-\delta)S_0 = (b-c) + a\delta\left[W_0 + J_0 - S_0\right]. \tag{18}$$

The key to proving that the equilibrium of this model is unique is to show that there exists a unique S_0 that solves equation (18). To show this and to derive the comparative static results that we discuss in the next section, it is useful to first show that the gains to continuing an employment relationship $(Z - S)$ are decreasing in the joint value of search S. For this purpose, we now establish the following lemma.

Lemma 1 *The effect of a marginal increase in the joint value of search S on the match value Z is given by*

$$\frac{\mathrm{d}Z}{\mathrm{d}S} = \frac{\delta\lambda m}{(1-\delta) + \delta\lambda m} < 1, \tag{19}$$

---

[13]For this to be the case it is sufficient for $(b-c)$ and or p to be sufficiently large.

where

$$m \equiv 1 - \frac{1}{2}\left[\mathrm{E}_{\rho}\left[G(\widetilde{\gamma})\right]\left(1 + \frac{1}{1 + H'(\widetilde{\gamma})}\right) - \mathrm{E}_{\gamma}\left[1 - F(\widetilde{\rho})\right]\left(1 + \frac{1}{1 - K'(\widetilde{\rho})}\right)\right]. \quad (20)$$

Having established this result, we can now prove that the equilibrium of our model is unique.

Proposition 1 *In the model without self-imposed rigidities, there exists a unique equilibrium.*

4.4 The Effects of a Reduction in the Firing Tax

We are interested in the effect of a reduction in the firing tax ϕ on the equilibrium of the economy derived in the previous subsections. In this section our primary interest is how such a reduction affects the well being of workers and the profits of firms. We will argue that a reduction in the firing tax leads to more aggressive bargaining behavior between firms and workers which, in turn, leads to an increase in job instability. We show that because of the increase in job instability some groups in society and, under some circumstances society as a whole, can be made worse off.

The firing tax only affects the match value Z through the joint value of search S. This follows immediately from the definitions of Z and Z^e in (7) and (15) which together imply that

$$\frac{\mathrm{d}Z}{\mathrm{d}\phi} = \frac{\delta\lambda}{1 - \delta(1 - \lambda)}\frac{\mathrm{d}Z^e}{\mathrm{d}S}\frac{\mathrm{d}S}{\mathrm{d}\phi}. \quad (21)$$

The next lemma shows that the last term on the right hand side of (21), $\mathrm{d}S/\mathrm{d}\phi$, is negative, that is, that a reduction in the firing tax increases the joint value of search. Essentially, such a reduction makes it less costly for firms and workers to separate, simply because alternative trading partners can be found at lower cost.

Lemma 2 *A reduction in the firing tax ϕ increases the expected joint value of separation for a firm-worker pair S, i.e. $\mathrm{d}S/\mathrm{d}\phi < 0$.*

It then follows from (21) that, in order to determine the effect of a change in the firing tax ϕ on Z, it is sufficient to study the sign of $\mathrm{d}Z^e/\mathrm{d}S$. Using (A-1) and (A-2) in the proof of Lemma 1 we have that

$$\frac{\mathrm{d}Z^e}{\mathrm{d}S} = \frac{1 - \delta + \delta\lambda}{1 - \delta + (1 - q)\delta\lambda} \left(\frac{\partial Z^e}{\partial S} + \frac{\partial Z^e}{\partial \widetilde{\gamma}} \frac{\mathrm{d}\widetilde{\gamma}}{\mathrm{d}S} + \frac{\partial Z^e}{\partial \widetilde{\rho}} \frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}S} \right), \tag{22}$$

where $q = \left(\mathrm{E}_\rho[G(\widetilde{\gamma})] + \mathrm{E}_\gamma[1 - F(\widetilde{\rho})] \right)/2$ is the expected probability of agreement. Since $q \in [0, 1]$, the first term in the right hand side of (22) is positive which implies that the sign of $\mathrm{d}Z^e/\mathrm{d}S$ is determined by the three terms in the round brackets. Equation (A-4) in the proof of Lemma 1 shows that the first term is given by

$$\frac{\partial Z^e}{\partial S} = 1 - q \geq 0.$$

Intuitively, whenever the firm and the worker disagree, which happens with probability $(1 - q)$, they obtain a higher payoff the higher is S. For given $\widetilde{\gamma}$ and $\widetilde{\rho}$ his effect is unambiguously positive and represents the marginal benefit of an increase in S.

The second and the third terms in brackets in (22) represent the marginal cost of an increase in S. In particular, from the proof of Lemma 1 we know that

$$\frac{\partial Z^e}{\partial \widetilde{\gamma}} \frac{\mathrm{d}\widetilde{\gamma}}{\mathrm{d}S} = -\frac{E_\rho[G(\widetilde{\gamma})]}{2(1 + H'(\widetilde{\gamma}))} \left[1 - \frac{\mathrm{d}Z}{\mathrm{d}S} \right] \leq 0$$

and

$$\frac{\partial Z^e}{\partial \widetilde{\rho}} \frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}S} = -\frac{E_\gamma[1 - F(\widetilde{\rho})]}{2(1 - K'(\widetilde{\rho}))} \left[1 - \frac{\mathrm{d}Z}{\mathrm{d}S} \right] \leq 0.$$

Intuitively, an increase in S leads to more aggressive bargaining behavior and increases the probability of separation, i.e. it reduces $\widetilde{\gamma}$ and increases $\widetilde{\rho}$. This increase in the

probability of separation, in turn, has a negative effect on the match value of a worker-firm pair that has experienced a productivity shock , i.e. it reduces Z^e. It is important to note that the fact that in this model an increase in the probability of separation has a negative effect on the match value is due to firms and workers separating 'too often' which in turn is due to the negotiations being hampered by the presence of private information. In a standard model in which negotiations over compensation always take place under public information, an increase in S also leads to more job instability; however, there such an increase in job instability is *not costly* for the firm and the worker, since the employment relationships that are discontinued have no value.

To see when a reduction in the firing tax can actually make a worker-firm pair worse off, consider the following lemma.

Lemma 3 *(a) Suppose that the distributions $G(\gamma)$ and $F(\rho)$ are such that for some $\kappa < \infty$*

$$\min \left\{ \lim_{\gamma \to \sup \Gamma} H'(\gamma), - \lim_{\rho \to \inf P} K'(\rho) \right\} \leq \kappa.$$

Then there exists an $\overline{S}$ and an $\underline{S} \leq \overline{S}$ such that

$$\frac{\mathrm{d}Z}{\mathrm{d}S} = \begin{cases} \leq 0 & \text{if } S < \underline{S} \\ > 0 & \text{if } S > \overline{S}. \end{cases}$$

If the distributions do not satisfy the foregoing condition, $\underline{S}$ might not (but still can) exist. For $\underline{S} \leq S \leq \overline{S}$, the sign of $\mathrm{d}Z/\mathrm{d}S$ is ambiguous and depends on the precise distributions of ρ and γ.

(b) Z reaches its global maximum to the right of $\overline{S}$.

This lemma is key for all the results in the paper. To get a sense for the condition on the distributions in part (a), note first that it rules out situations in which $f(\underline{\rho}) = g(\overline{\gamma}) = 0$, $f'(\underline{\rho}) > 0$ and $g'(\overline{\gamma}) < 0$, where $\overline{\gamma} = \sup \Gamma$ and $\underline{\rho} = \inf P$. The condition

is satisfied, for instance, for truncated exponential, normal and Pareto distributions as well as uniform distributions. To focus on the most interesting case, we assume for the rest of the discussion that this condition is indeed satisfied.

The key insight of the lemma is that the negative effect of an increase in S dominates when S is small and the positive effect dominates when S is large. For intermediate values of S, the effect of an increase in S on Z is ambiguous and depends on the particular distributions of γ and ρ. The intuition for this result is as follows: when S is initially small, it is very costly for firms and workers to separate and search for alternative trading partners. As a result the firm and the worker do not bargain very aggressively with each other and separations are rare in equilibrium. Thus, the marginal benefit of an increase in S deriving from the higher payoff that the worker and the firm obtain when they separate is relatively small and is dominated by the marginal cost deriving from the increased probability of inefficient separation. When S is initially large, however, it is not very costly for firms and workers to separate. They therefore bargain aggressively and separate fairly often. In this case the marginal benefit of an increase in S is large and dominates the marginal cost. Finally, we note that, although an increase in S can have a negative effect on Z for low initial values of S, the global maximum of Z is achieved at the upper limit of S.

Recall from Lemma 2 that S is decreasing in ϕ. Thus, together Lemmas 2 and 3 imply that a small reduction in firing taxes makes worker-firm pairs worse off if firing taxes are initially high and better off if firing taxes are initially low. To formally state this and other results, it is useful to introduce the following definition.

Definition 1 *Let $\underline{\phi}$ denote the unique ϕ that solves $S(\phi) = \overline{S}$, where $S(\phi)$ is the equilibrium value of S given ϕ. Similarly, let $\overline{\phi}$ denote the unique ϕ that solves $S(\phi) = \underline{S}$.*

Note that the existence and uniqueness of $\underline{\phi}$ and $\overline{\phi}$ follow from Lemma 2. We can now state the following proposition which summarizes the first set of results in this section.

Proposition 2 (Comparative statics for insiders)

(a) The effect of a marginal change in ϕ on Z is

$$\frac{dZ}{d\phi} = \begin{cases} \geq 0 & if \ \phi > \overline{\phi} \\ < 0 & if \ \phi < \underline{\phi}. \end{cases}$$

(b) The effect of a marginal change in ϕ on W_0 and J_0 is

$$\frac{dW_0}{d\phi} = \frac{dJ_0}{d\phi} = \begin{cases} \geq 0 & if \ \phi > \overline{\phi} \\ < 0 & if \ \phi < \underline{\phi}. \end{cases}$$

(c) The effect of a marginal change in ϕ on W_1 and J_1 is

$$\frac{dW_1}{d\phi} \geq 0 \ \ if \ \ \phi > \overline{\phi} \ \ \ and \ \ \ \frac{dJ_1}{d\phi} \leq 0 \ \ for \ all \ \phi.$$

Part (a) of the proposition formally describes the effect of a reduction in the firing tax on the match value, that was discussed above, and parts (b) and (c) investigate its effect on the distribution of the match value. Part (b) shows that a reduction in firing taxes has an adverse effect on firms and workers at the beginning of an employment relationship if firing taxes are initially high and a positive effect if they are initially low. This is again due to the two countervailing effects of a reduction in the firing tax, i.e. a reduction in the cost and an increase in the probability of separation. Finally, part (c) describes the effect of a reduction in the firing tax on firms and workers during an employment relationship. It shows that such a reduction always increases J_1 since it improves the firm's bargaining position relative to that of the worker. A reduction in firing taxes can also be beneficial for the worker when these taxes are initially low

22

because, although it weakens the worker's bargaining position, it increases the value of the match over which bargaining takes place.

Having discussed the effect of a reduction in the firing tax on workers and firms that are in an employment relationship, we now turn to its effect on workers and firms that are not in an employment relationship.

Proposition 3 (Comparative Statics Outsiders)

The effects of a marginal change in ϕ on U and V satisfy are

$$\frac{\mathrm{d}U}{\mathrm{d}\phi} = \frac{\mathrm{d}V}{\mathrm{d}\phi} = \frac{1}{2}\frac{\mathrm{d}S_0}{\mathrm{d}\phi} = \begin{cases} \geq 0 & \text{if } \phi > \overline{\phi} \\ < 0 & \text{if } \phi < \underline{\phi}. \end{cases}$$

For $\underline{\phi} \leq \phi \leq \overline{\phi}$, the sign of $dU/d\phi$ and $dV/d\phi$ are ambiguous and depend on the distributions of γ and ρ.

Thus, even firms and workers who are currently searching for trading partners can be made worse off by a reduction in the firing tax. In particular a reduction in firing taxes when they are initially high reduces the match value Z without increasing the probability that firms and workers are matched. As a result, unemployed workers and firms with open vacancies are made worse off.

So far the analysis has shown that a reduction in firing taxes can have an adverse effect both on firms and workers who are already in an employment relationship and on those who are still searching for potential trading partners. Indeed, Propositions 2 and 3 show that this reduction can make all firms and workers worse off and thus reduce overall welfare, as shown in the following proposition.

Proposition 4 *Assume a utilitarian welfare function in which welfare is given by the sum of the expected utilities of all firms and workers. Then a small reduction in the firing tax reduces welfare when labor market frictions are high, i.e. when $\phi \geq \overline{\phi}$.*

In summary, this section makes two main points. First, it shows that when firms and workers bargain under asymmetric information, a reduction in the costs of separation (e.g. a reduction in firing taxes) leads to more job instability. Second, it shows that because of the increase in job instability, a small reduction in separation costs can make some groups in society worse off and, when separation costs are initially high, reduce overall welfare. The adverse effects of a small reduction in separation costs are, however, dominated by its positive effects when separation costs are initially low. This suggests that labor market reforms may be harder to implement in continental Europe, where labor market frictions are high, than in the UK or the US, where frictions are already less severe.

It is also important to note that in our model a sufficiently large reduction in labor market frictions is always welfare enhancing. This suggests that in some situations it may be preferable to implement large reforms rather than piece-meal reforms.

Throughout our analysis we have assumed that the firing tax ϕ has to be paid regardless of whether separation is caused by the firm rejecting the worker's offer or by the worker rejecting the firm's offer. An alternative would be to assume that the cost of the firing tax is incurred only if the firm rejects the worker's offer and, in this sense, lays off the worker. If this were the case the effects that we discussed above still arise albeit in a weaker form. In particular, in such a model a reduction in the firing tax still increases the outside options of firms and thus induces them to bargain more aggressively. The effect on the worker's bargaining behavior, however, is different. In particular, if a reduction in the firing tax leads to a reduction in the match value, then it actually leads to less aggressive bargaining behavior by the worker. This is the case since a reduction in the match value reduces the value of being unemployed and thus increases the worker's cost of having his offer rejected. For a reduction in the firing tax

to be welfare reducing, it must therefore be the case that the more aggressive bargaining by firms outweighs the less aggressive bargaining by workers. Whether or not this is the case, depends on a number of factors, including the probabilities with which each side makes offers.

4.5   Free Entry

Before we move on to analyze self-imposed rigidities in the next section, we briefly digress to discuss free entry by firms. A more formal discussion, including a version of our model with free entry, is provided in the Appendix, where we show that the main insights of the analysis above continue to hold.

Although with free entry the expected value of a vacancy, V, is always equal to zero in equilibrium, the outside option faced by a firm that needs to renegotiate the wage of one of its existing employees following a productivity shock still depends on the firing tax ϕ, because the firm cannot avoid paying this tax if the employment relationship is dissolved. Therefore, also in the presence of free entry by firms it is still the case that a reduction in the firing tax increases the joint expected value of separation S and, through this channel, leads to more aggressive bargaining behavior and more job instability. As a result, a marginal reduction in the firing tax reduces the match value Z and the joint value of search S_0 when separation costs are initially high and it increases Z and S_0 when separation costs are initially low. Moreover, it continues to be the case that a marginal reduction in the firing tax reduces welfare when labor market frictions are high.

While the comparative statics on the match value and the joint outside options of a worker and firm that are in an employment relationship do not change as a consequence of the introduction of free entry, the comparative statics on the division of the surplus

at the beginning of an employment relationship do. This is the case since, with free entry, the value of a vacancy V is always equal to zero and thus unaffected by changes in ϕ. However, changes in ϕ have an effect on U. To see the implications of this, consider a reduction in ϕ when ϕ is initially low. This reduction increases the value of being unemployed U but does not change V. As a result, the bargaining position of unemployed workers is strengthened and thus firms may actually be made worse off.

5   Self-Imposed Rigidities

In the model that we analyzed in the previous sections, when firms and workers face productivity shocks they may fail to reach an agreement over the distribution of the adjustment costs although it would be efficient for them to reach such an agreement. To the extent that firms and workers are aware of this inefficiency in their future negotiations, one would expect them to take contractual actions at the beginning of their relationship to mitigate it. In this section we investigate this possibility. We first show that firms and workers may want to increase future separation costs so as to commit to less aggressive bargaining behavior in the future. In other words, firms and workers may want to complement the exogenous rigidities that they face with their own, self-imposed rigidities. We assume that firms and workers can commit themselves to these self-imposed rigidities at the beginning of their relationship even though in the future, if it becomes clear that they have to separate, they would prefer not to incur the extra cost. In the real world, firms and workers commit to different separation costs in a variety of ways. For instance, separation between a firm and a worker is less costly if the firm uses a fixed-term employment contract than if it uses a permanent contract. In this section we study the conditions under which firms and workers choose to commit to self-imposed rigidities and investigate the effects of a reduction in firing taxes on the

optimal level of these rigidities.

In particular, we now analyze the full-fledged model described in Section 2 in which a firm and a worker that have just been matched can contract over the extra cost R that the firm incurs in case the employment relationship is terminated in the future.[14] To simplify the exposition we now restrict attention to distributions for which $H''(\gamma)$ is not 'too positive' and $K''(\rho)$ is not 'too negative,' in the sense made clear in the following lemma. As the lemma shows, these conditions ensure that Z is everywhere convex in S.

Lemma 4 *Assume that the following conditions hold:* $H''(\cdot) < [1+H'(\cdot)][2+H'(\cdot)]/H(\cdot)$ *for all* $\gamma \in \Gamma$ *and* $K''(\cdot) > [1 - K'(\cdot)]K'(\cdot)/K(\cdot)$ *for all* $\rho \in P$. *Then* Z *is everywhere convex in* S, *i.e.* $\underline{S} = \overline{S}$.

These conditions are satisfied by a number of common distributions, including truncated exponential and Pareto distributions as well as uniform distributions. Slightly weaker results than those that we present in this section can be proven for more general distributions.

Suppose that a firm and a worker are matched and bargain over R. Since they are risk neutral and not liquidity constrained they agree on the level of R that maximizes their match value Z, as given in (7). Note that in a model in which bargaining over adjustment to productivity shocks took place under public information it would never be optimal for workers and firms to commit to a strictly positive R. However, as the following proposition shows, when bargaining takes place under private information, committing to higher separation costs may indeed be optimal.

---

[14]The results that we derive in this section would not be affected if the extra costs R are incurred not only by the firm, as in this specification, but also (in part or in full) by the worker.

Proposition 5 (Self-Imposed Rigidities) *There exists a unique $\widehat{\phi}$ such that*

$$R = \begin{cases} \overline{R} & \text{if } \phi \geq \widehat{\phi} \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\frac{\partial \widehat{\phi}}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \widehat{\phi}}{\partial p} > 0.$$

The intuition for this result is as follows. On the one hand, self-imposed rigidities make it more costly for the firm and the worker to separate, which happens with positive probability in equilibrium. On the other hand, however, precisely because these rigidities make separation more costly, they also make it less likely, since they induce less aggressive bargaining. When labor market frictions are initially high and/or workers are very productive, the benefit of self-imposed rigidities dominate the costs if labor market frictions are large and the opposite holds if labor market frictions are small and/or workers are not very productive.

The proposition provides a novel and intuitive explanation for why firms and workers may commit to higher future separation costs: since firms and workers anticipate that they may bargain 'too aggressively' in the future over the adjustment costs or benefits of shocks they may find it optimal to commit to high separation costs in order to make the employment relationship more stable. The proposition also shows that if self-imposed rigidities serve this purpose, then these self-imposed-rigidities *complement* exogenous rigidities: when labor market frictions are high, in the sense that ϕ is large, it is optimal for the firm and the worker to make it even more costly for themselves to separate in the future by setting R as high as possible. In contrast, when labor market frictions are low it is optimal for the firm and the worker to avoid any additional cost of separation by setting $R = 0$. This result is consistent with the observation that in countries such as Japan, in which labor market frictions are relatively high and firms face significant

28

government imposed rigidities, many firms voluntarily commit to additional constraints that make it even more costly for them to separate from their employees.[15]

Finally, we can use Proposition 5 to analyze the effect of a reduction in firing taxes on the employment contracts that firms and workers adopt. For this purpose, suppose that firing taxes are initially high, in the sense that $\phi > \widehat{\phi}$, and consider the effect of a reduction in the firing tax. If this reduction is sufficiently small then the firing tax ϕ remains above the cut-off level $\widehat{\phi}$ and firms and workers continue to adopt the most rigid employment contracts. However, if the reduction is sufficiently large, so that ϕ falls below $\widehat{\phi}$, firms and workers switch to the most flexible employment contracts, i.e. from $R = \bar{R}$ to $R = 0$. This switch leads to a discrete reduction in the cost of separation and thus induces the firms and workers to bargain more aggressively and to separate more often following productivity shocks. The instability caused by a reduction in firing taxes is therefore amplified by this endogenous contractual response to the original policy change.

We note that the results in this section still hold if we allow for free entry of firms along the lines of the model discussed in Section 4.5 and in the Appendix. This is the case because the adoption of contractual rigidities discussed in the present section depends on the joint match value Z and not on how this value is distributed between the worker and the firm. As discussed in Section 4.5, the introduction of free entry does not affect the comparative statics on the joint match value Z and thus does not affect our conclusions about the implications of changes in firing taxes for self-imposed rigidities.

---

[15]See, for instance, Dore (1996) who states that *"What was not always recognized [...] was the importance of self-imposed rigidities, most fully exemplified in the Japanese firm. By self-imposed rigidities, I mean the acceptance, by managers, of a wide range of constraints on their freedom of action - lifetime employment guarantees, tight seniority constraints on promotion, acceptance of the need to engineer consent, to maintain close consultation with employees or their unions [...]."*

6 Efficient Bargaining

The model introduced in Section 3 assumed that when the firm and the worker renegotiate the wage, they play a simple bargaining game in which each side makes a take-it-or-leave-it offer with probability 1/2. We assumed this particular bargaining game because of its simplicity and because it is consistent with the standard assumption, which we also adopted, that the firm and the worker Nash bargain over the wage when they are first matched. A potential concern, however, is that the results that we derived are not robust and depend crucially on the assumed bargaining game. In this section we address this concern. In particular, we change the model from Section 2 in two ways. First, to simplify the exposition, we assume that each employment relationship is hit by a shock at most once. Second, we now assume that, of all the possible bargaining games that the firm and the worker could play when the shock hits, they play the most efficient, voluntary bargaining game, i.e. the bargaining game that maximizes the expected match value Z^e, subject to the interim participation constraints.[16] In other words, we assume that at the renegotiation stage, the firm and the worker play the Myerson and Satterthwaite (1983) bargaining game. This assumption addresses the robustness concern since the firm and the worker would commit to exactly this renegotiation bargaining game if they could contract over it when they are first matched. Also, by assuming that the firm and the worker adopt the most efficient bargaining game we are focusing on the minimum inefficiency that has to arise in a labor market in which wage renegotiations take place in the presence of private information.

It should be stressed that the approach that we adopt in this section is very general since we are allowing for any possible bargaining game including, for instance, alternat-

---

[16]The interim participation constraints ensure that the firm and the worker are willing to participate in the bargaining game after they have learned the realization of p and γ respectively.

ing offers games. The only restrictive assumption that we maintain is that the firm and the worker can commit to the outcome of the renegotiation game, i.e. they can commit not to renegotiate again. Thus, if the firm and the worker disagree on a wage they separate even if it is commonly known that there are still gains from trade. It is important to note that if the firm and the worker were unable to commit to separate if they disagree on a wage, the bargaining inefficiency that we focus on would necessarily be even larger. Thus, by assuming that the firm and the worker are able to commit to separate in the case of disagreement we ensure that we are analyzing the minimum inefficiency that has to arise in a labor market in which wage renegotiations take place in the presence of private information.[17]

In the formal analysis, which we have relegated to the appendix, we start by following Myerson and Satterthwaite (1983) to describe the most efficient, voluntary bargaining game. In particular, we adopt the mechanism design approach to describe the bargaining game that maximizes Z^e subject to the participation constraints $W(\gamma) \geq U$ and $J(\rho) \geq V - \phi - R$, where $W(\gamma)$ is the worker's expected payoff from playing the bargaining game given that his costs are γ and $J(\rho)$ is the firm's expected payoff from playing the bargaining game given that its benefit is ρ (see Lemma A3 in the Appendix). We then observe that as long as continuation of the employment relationship is not always optimal, bargaining between the firm and the worker is inefficient, in the sense that they sometimes separate although it would be optimal for them to continue their relationship. Next we establish the existence and uniqueness of the equilibrium (see Proposition A4) and then turn to the comparative statics. We argue that, just as in the main model, a liberalization of the labor market makes it less costly for a firm and

---

[17]Note also that any public information model in which wages are determined by take-it-or-leave-it offers makes the same assumption, i.e. it assumes that if an offer is rejected, the firm and the worker separate although it is commonly known that there are gains from trade (see, for instance, Pissarides 2000).

a worker to separate. We show that this again has an ambiguous effect on the match value since, on the one hand, it increases the payoff they realize in case they separate but, on the other hand, also makes separation more likely. The only difference to the analysis of the main model is that we are no longer able to characterize analytically when either effect dominates in the case of general distributions. However, we can show that our results continue to hold for a number of common distributions, in particular the standard normal, standard exponential, and standard uniform distributions. We argue that for these distributions our main results, i.e. Propositions 2 to 5, continue to go through.

As an illustration, consider a simple numerical example in which γ and ρ are uniformly distributed on $[0,1]$, $b = c = 0$, $a = 1/2$, $p = \lambda = 1/10$, $d = 5/100$, $r = 5/90$ and $\delta = 9/10$. In this example, the optimal mechanism ensures that a firm-worker pair that has been hit by a shock agrees to continue their relationship with probability

$$
q(\gamma, \rho) = \begin{cases} 1 & \text{if } p - \gamma \geq \frac{1+\mu}{1+2\mu}\left(S_0 - \phi - 1\right) + \frac{\mu}{1+2\mu} \\ 0 & \text{otherwise,} \end{cases}
$$

where $\mu \in [0, \infty)$. For $(S_0 - \phi - 1) \leq -1$, $\mu = 0$ and for $(S_0 - \phi - 1) > -1$, $\mu > 0$ solves the interim participation constraint

$$
E_{\gamma, \rho}\left[\left(2\left(\rho - \gamma\right) + S_0 - \phi - 2\right) q(\gamma, \rho)\right] = 0.
$$

In the appendix we show how to work out the values of all the endogenous variables for $\phi = \{0, 0.1, ..., 1\}$. The results are represented in Figures 2 and 3. In line with Proposition 4, Figure 2 shows that a reduction in the firing tax reduces welfare when the labor market frictions are high and increases it when they are low. The figures also show that similar comparative statics hold for W_0, J_0, U and V, consistent with Propositions 2 and 3. Finally, in line with Proposition 2, Figure 3 shows that a reduction in the

firing tax leads to an increase in J_1 and a reduction in W_1.

7   Conclusions

When firms and workers bargain over compensation they are often asymmetrically informed about relevant payoffs, such as profits and the workers' opportunity costs. It is well-known that in such situations bargaining is likely to be inefficient, in the sense that firms and workers sometimes fail to realize all gains from trade. In this paper we investigate the implications of this type of bargaining inefficiency for the functioning of labor markets.

Our analysis provides three main insights. First, it shows that when bargaining between firms and workers takes place under asymmetric information, a small reduction in separation costs can make some groups in society worse off and, as a result, reduce welfare. Essentially, this can happen since such a reduction induces firms and workers to bargain more aggressively and therefore makes employment relationships more instable. This instability effect can dominate the direct benefit of a reduction in separation costs so that firms and workers can be made worse off. Second, our analysis shows that when firms and workers anticipate that future bargaining over adjustment to productivity shocks will take place under asymmetric information and may therefore be inefficient, they have an incentive at the beginning of their employment relationship to commit to rigid employment contracts that make it more costly for them to separate in the future. Finally, our analysis adds to our understanding of the possible interactions between exogenous changes in the labor market environment – owing to policy reform, technological change or globalization – and the endogenous contractual response of workers and firms.[18] In particular, we show that significant reductions in

---

[18]We note that, although throughout the paper we conducted our comparative statics analysis focus-

labor market frictions may lead to a one-off switch from rigid to flexible employment contracts. This endogenous contractual change induces firms and workers to bargain even more aggressively and thus further destabilizes employment relationships.

As regards the robustness of our conclusions to alternative modelling choices, we address possible concerns that our results may depend crucially on the assumed bargaining game by allowing firms and workers to play the most efficient bargaining game and show that for a number of common distributions our results continue to hold. We instead maintain throughout our analysis the simplifying assumption that productivity shocks are match-specific and thus uncorrelated across time and individual workers or firms. We believe that it would be interesting to analyze a model in which shocks are correlated – as this would shed some light on the, possibly different, effects of changes in separation costs on high and low productivity workers and firms – and we leave this analysis for future work.

---

ing on a reduction in the firing tax, ϕ, improvements in the search technology (i.e. an increase in a) would yield very similar results. For details see the working paper version of this paper (Matouschek, Ramezzana and Robert-Nicoud 2006).

Appendix

This appendix is divided into three sections. In Section A we prove all lemmas and propositions stated in the main text; in Section B we extend our model by allowing for free entry by firms whilst in Section C we allow workers and firms to choose the most efficient game when bargaining over adjustment to productivity shocks.

A   Proofs of Lemmas and Propositions

Proof of Lemma 1: Differentiating (7) gives

$$\frac{\mathrm{d}Z}{\mathrm{d}S} = \frac{\delta\lambda}{(1-\delta) + \delta\lambda} \frac{\mathrm{d}Z^e}{\mathrm{d}S}, \tag{A-1}$$

where Z^e is the expected match value when a shock hits. To obtain $\mathrm{d}Z^e/\mathrm{d}S$ we totally differentiate (15):

$$\frac{\mathrm{d}Z^e}{\mathrm{d}S} = \frac{\partial Z^e}{\partial S} + \frac{\partial Z^e}{\partial\tilde{\gamma}} \frac{\mathrm{d}\tilde{\gamma}}{\mathrm{d}S} + \frac{\partial Z^e}{\partial\tilde{\rho}} \frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}S} + \frac{\partial Z^e}{\partial Z} \frac{\mathrm{d}Z}{\mathrm{d}S}. \tag{A-2}$$

Using this expression to substitute for $\mathrm{d}Z^e/\mathrm{d}S$ in (A-1) and solving for $\mathrm{d}Z/\mathrm{d}S$ gives

$$\frac{\mathrm{d}Z}{\mathrm{d}S} = \frac{\delta\lambda}{(1-\delta) + \delta\lambda\left(1 - \partial Z^e/\partial Z\right)} \left(\frac{\partial Z^e}{\partial S} + \frac{\partial Z^e}{\partial\tilde{\gamma}} \frac{\mathrm{d}\tilde{\gamma}}{\mathrm{d}S} + \frac{\partial Z^e}{\partial\tilde{\rho}} \frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}S} \right). \tag{A-3}$$

This expression reduces to that stated in the lemma. To show this, we first partially differentiate (15) with respect to Z and S to obtain

$$\frac{\partial Z^e}{\partial Z} = 1 - \frac{\partial Z^e}{\partial S} = \frac{1}{2} \left(E_\rho[G(\tilde{\gamma})] + E_\gamma[1 - F(\tilde{\rho})] \right). \tag{A-4}$$

Next, partially differentiating (15) with respect to $\tilde{\gamma}$ and $\tilde{\rho}$ and using the first order conditions for $\tilde{\gamma}$ and $\tilde{\rho}$ given by (10) and (13) we have that

$$\frac{\partial Z^e}{\partial\tilde{\gamma}} = \frac{1}{2} E_\rho\left[G(\tilde{\gamma})\right] \quad \text{and} \quad \frac{\partial Z^e}{\partial\tilde{\rho}} = -\frac{1}{2} E_\gamma\left[1 - F(\tilde{\rho})\right]. \tag{A-5}$$

Finally, we totally differentiate (10) and (13) to get

$$\frac{\mathrm{d}\widetilde{\gamma}}{\mathrm{d}S} = -\frac{1}{1 + H'(\widetilde{\gamma})}\left[1 - \frac{\mathrm{d}Z}{\mathrm{d}S}\right] \quad \text{and} \quad \frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}S} = \frac{1}{1 - K'(\widetilde{\rho})}\left[1 - \frac{\mathrm{d}Z}{\mathrm{d}S}\right]. \tag{A-6}$$

Substituting (A-4) - (A-6) into (A-3) and solving for $\mathrm{d}Z/\mathrm{d}S$ then gives (19). The fact that $\mathrm{d}Z/\mathrm{d}S < 1$ then follows from the assumptions that $H'(\widetilde{\gamma}) > 0$ and $K'(\widetilde{\rho}) < 0$, which ensure that $m \in [-1, 1]$ holds for all $\widetilde{\gamma}(\rho) \in \Gamma$ and all $\widetilde{\rho}(\gamma) \in \mathrm{P}$, and that $\lambda < (1-\delta)/(2\delta)$. ∎

Proof of Proposition 1: As discussed in the main text, we assume that the parameters of the model are such that equilibria with $U > 0$ and $V > 0$ always exist and we focus on these equilibria. However, we note that even if these conditions on the parameters did not hold, trivial equilibria with neither party participating in the market always exist.

Noting that $S = S_0 - R - \phi$ and, from (6), that $W_0 + J_0 = Z$ we can write (18) as

$$(1 - \delta)(S + R + \phi) = (b - c) + a\delta(Z - S - R - \phi). \tag{A-7}$$

Lemma 1 implies that $\mathrm{d}Z/\mathrm{d}S < 1$ and thus that the right hand side of (A-7) is decreasing in S. Since the left hand side of this equation is increasing in S there can only exist a unique equilibrium value for S and thus for S_0. From (18), a unique equilibrium value for S_0 implies a unique equilibrium value for $(Z - S_0)$. From (10) and (13), a unique equilibrium value for $(Z - S_0)$ implies unique equilibrium values for $\widetilde{\gamma}$ and $\widetilde{\rho}$. From (1), (2), (16) and (17) a unique equilibrium value for $(Z - S_0)$ implies unique equilibrium values for U and V. ∎

Proof of Lemma 2: Using $S = S_0 - \phi - R$ and totally differentiating (18) gives

$$\frac{dS}{d\phi} = -\frac{1 - \delta + a\delta}{1 - \delta + a\delta[1 - dZ/dS]} < 0, \tag{A-8}$$

where the inequality follows from $dZ/dS < 1$ as shown in Lemma 1. ∎

Proof of Lemma 3: We make use of the following facts: (i) $dZ/dS = m/\alpha(m)$, where m is given by (20) and $\alpha \equiv (1 - \delta + \delta\lambda m)/(\delta\lambda) > 1$ by $m \in [-1, 1]$. (ii) By (20), we have $m = 1 - E_\rho[G(\widetilde{\gamma})][2 + H'(\widetilde{\gamma})]/[2 + 2H'(\widetilde{\gamma})] - E_\gamma[(1 - F(\widetilde{\rho})][2 - K'(\widetilde{\rho})]/[2 - 2K'(\widetilde{\rho})]$. (iii) By the monotone hazard rate conditions, $[2 + H'(\widetilde{\gamma})]/[1 + H'(\widetilde{\gamma})] \in [1, 2]$ and $[2 - K'(\widetilde{\rho})]/[1 - K'(\widetilde{\rho})] \in [1, 2]$. (iv) By (10) $\widetilde{\gamma}$ is decreasing in S and by (13) $\widetilde{\rho}$ is increasing in S.

(a) We start with the second inequality. Fact (iv) implies that for any $\varepsilon > 0$, there exists an $\overline{S}(\varepsilon)$ such that $\forall S > \overline{S}(\varepsilon)$, $G(\widetilde{\gamma}) < \varepsilon$ and $1 - F(\widetilde{\rho}) < \varepsilon$. Together with facts (ii) and (iii), we get $m > 1 - 2\varepsilon$. Invoking fact (i), this in turn implies

$$\frac{dZ}{dS} = \frac{m}{\alpha} > \frac{1 - 2\varepsilon}{\alpha},$$

which is strictly positive if ε is chosen adequately. Thus by continuity of the supports Γ and P an $\overline{S}$, as defined in the lemma, exists.

Consider next the first inequality. Fact (iv) implies that for any $\xi > 0$ there exists an $\underline{S}(\xi)$ such that $G(\widetilde{\gamma}) > 1 - \xi$ and $1 - F(\widetilde{\rho}) > 1 - \xi$. Next, let us impose our assumption $\min\{\lim_{\gamma \to \sup \Gamma} H'(\gamma), -\lim_{\rho \to \inf P} K'(\rho)\} \leq \kappa$, where κ is some finite real number. In this case, using facts (i), (ii) and (iii), we obtain $m < -[1 - \xi(2 + \kappa)]/(1 + \kappa)$. Invoking fact (i), this in turn implies

$$\frac{dZ}{dS} = \frac{m}{\alpha} < -\frac{1 - \xi(2 + \kappa)}{(1 + \kappa)\alpha}$$

which is strictly negative if ξ is chosen small enough. Thus by continuity of the supports Γ and P an $\underline{S}$, as defined in the lemma, exists. If our assumption above fails, that is, if $\kappa \to \infty$, then such an $\underline{S}$ *might* not exist.

(b) By part (a), Z grows unbounded as S increases since $dZ/dS > 0$ if S is large enough. Moreover, by the fact that $H(\cdot)$ and $K(\cdot)$ are continuously differentiable Z is also a continuously differentiable function of S. The result thus immediately follows. ■

Proof of Proposition 2: Part (a) follows immediately from Lemmas 1 and 2.

For part (b) note first that from (16) and (17), $(1 - \delta)dU = a\delta d(W_0 - U)$ and $(1-\delta)dV = a\delta d(J_0 - V)$. Next, from (1) and (2) we have $W_0 - U = J_0 - V$. Substituting $W_0 - U$ for $J_0 - V$ in the foregoing equations then shows that $dU = dV$ and hence $dU/d\phi = dV/d\phi$. Differentiating (1) and (2) and substituting $dV/d\phi$ for $dU/d\phi$ then gives

$$\frac{dW_0}{d\phi} = \frac{dJ_0}{d\phi} = \frac{1}{2}\frac{dZ}{d\phi}.$$

The signs of $dW_0/d\phi$ and $dJ_0/d\phi$ follow readily from part (a).

For part (c) we differentiate (4) and (5) and substitute $dV/d\phi$ for $dU/d\phi$ to obtain

$$\frac{dW_1}{d\phi} = \frac{1}{2}\left[\frac{dZ}{d\phi} + 1\right] \quad \text{and} \quad \frac{dJ_1}{d\phi} = \frac{1}{2}\left[\frac{dZ}{d\phi} - 1\right].$$

Since ϕ affects Z via S only, we have

$$\frac{dZ}{d\phi} = \frac{dZ}{dS}\frac{dS}{d\phi}$$

which is negative (respectively positive) if and only if $dZ/dS > 0$ (respectively $dZ/dS < 0$) by the proof of Lemma 2. Therefore, a sufficient condition for $dW_1/d\phi$ to be positive is that $\phi > \overline{\phi}$, as claimed in the text. Using (A-8), we obtain

$$\frac{dJ_1}{d\phi} = -\frac{1}{2}\frac{a\delta + (1 - \delta)\left[1 + dZ/dS\right]}{1 - \delta + a\delta[1 - dZ/dS]}$$

which is negative since $1 > \partial Z / \partial S > -1$. ∎

Proof of Proposition 3: In the proof of Proposition 2 we have shown that $dU/d\phi = dV/d\phi$. We can therefore focus on the effect of a labor market liberalization on $S_0 = U + V$. Using $S = S_0 - \phi$ and (A-8) we have that

$$\frac{dS_0}{d\phi} = -\frac{a\delta dZ/dS}{1 - \delta + a\delta[1 - dZ/dS]}.$$

Thus, sign $\{dS_0/d\phi\} = -\text{sign}\,\{dZ/dS\}$. From Lemma 3 it then follows that $dS_0/d\phi \geq 0$ if $S < \underline{S}$ and $dS_0/d\phi > 0$ if $S > \overline{S}$. The signs of $dU/d\phi$ and $dV/d\phi$ then follow from the definition of $\overline{\phi}(a)$ and $\underline{\phi}(a)$. ∎

Lemma A1: *(a) In the model presented in Section 3 the unemployment rate is given by*

$$u = \left[1 + \frac{2a}{2\delta + \lambda\left\{E_\rho[1 - G(\widetilde{\gamma})] + E_\gamma[F(\widetilde{\rho})]\right\}}\right]^{-1}. \tag{A-9}$$

(b) The unemployment rate increases as the firing tax decreases, namely $du/d\phi < 0$ for any ϕ.

Proof: (a) This rate is computed as follows. The dynamics of the number of unemployed workers is $N_u(t+1) - N_u(t) = n - (a+\delta)N_u(t) + \lambda N_e(t)\{E_\rho[1 - G(\widetilde{\gamma})] + E_\gamma[F(\widetilde{\rho})]\}/2$ and the dynamics of the number of employed workers is given by $N_e(t+1) - N_e(t) = aN_u(t) - (\delta + \lambda\{E_\rho[1 - G(\widetilde{\gamma})] + E_\gamma[F(\widetilde{\rho})]\}/2) N_e(t)$. In steady state $N_u(t+1) = N_u(t) = N_u$ and $N_e(t+1) = N_e(t) = N_e$ and thus the total number of workers in the steady state is $N = N_u + N_e = n/\delta$ and the unemployment rate $u \equiv N_u/N$ is as given in (A-9).

(b) From (10) and (13), $\widetilde{\gamma}$ decreases and and $\widetilde{\rho}$ increases when S increases. The result then follows from the fact that $dS/d\phi < 0$, as shown in Lemma 2. ∎

Proof of Proposition 4: In steady state, welfare is given by $uS_0/2 + (1-u)Z/2$. Given that $\phi \geq \overline{\phi}(a)$ we know by Proposition 3 that when ϕ decreases, unmatched firms and workers and matched worker-firm pairs are both made worse off. Also, from Lemma A1, $\mathrm{d}u/\mathrm{d}\phi < 0$ for any ϕ. Since $Z > S_0$ it follows that those who lose their jobs and close the job position as a result of ϕ going down are worse off, too. Then the net result of a marginal reduction in the firing tax on welfare is unambiguously negative when labor market frictions are high to start with. ∎

Proof of Lemma 4: Assume that the following hold: $H''(\cdot) < [1+H'(\cdot)][2+H'(\cdot)]/H(\cdot)$ for all γ and that $K''(\cdot) > [1-K'(\cdot)]K'(\cdot)/K(\cdot)$ for all ρ. Next, consider

$$
\begin{aligned}
\frac{\mathrm{d}^2 Z}{\mathrm{d}S^2} = &\left[1 - \frac{\mathrm{d}Z}{\mathrm{d}S}\right] \frac{\delta\lambda}{2(1-\delta(1-\lambda m))} \times \\
&\left[\int_{\rho \in \mathrm{P}} \left\{\frac{H(\widetilde{\gamma})H''(\widetilde{\gamma})}{(1+H'(\widetilde{\gamma}))^2} - \frac{2+H'(\widetilde{\gamma})}{1+H'(\widetilde{\gamma})}\right\} \frac{\mathrm{d}\widetilde{\gamma}}{\mathrm{d}S} g(\widetilde{\gamma})\mathrm{d}F(\rho) \right. \\
&\left. + \int_{\gamma \in \Gamma} \left\{\frac{K(\widetilde{\rho})K''(\widetilde{\rho})}{(1-K'(\widetilde{\rho}))^2} - \frac{K'(\widetilde{\rho})}{1-K'(\widetilde{\rho})}\right\} \frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}S} f(\widetilde{\rho})\mathrm{d}G(\gamma)\right].
\end{aligned}
$$

Since $\mathrm{d}Z/\mathrm{d}S < 1$, $\mathrm{d}\widetilde{\gamma}/\mathrm{d}S < 0$ and $\mathrm{d}\widetilde{\rho}/\mathrm{d}S > 0$, we have that Z is convex in S for all S as long as $H''(\gamma)$ and $K''(\rho)$ obey the restrictions above. ∎

Proof of Proposition 5: We first argue that Z is quasi-convex in R. To see this note first that Z depends on R only through S. Thus, $\mathrm{d}Z/\mathrm{d}R = (\partial Z/\partial S)(\mathrm{d}S/\mathrm{d}R)$. Next note that a change in ϕ has the same effect on S as a change in R, i.e. $\mathrm{d}S/\mathrm{d}\phi = \mathrm{d}S/\mathrm{d}R$. Since we have shown in Lemma 2 that $\mathrm{d}S/\mathrm{d}\phi < 0$ it follows that $\mathrm{d}S/\mathrm{d}R < 0$. Thus, $\mathrm{d}Z/\mathrm{d}R = 0$ if and only if $\partial Z/\partial S = 0$. This, in turn, implies that at any R for which $\mathrm{d}Z/\mathrm{d}R = 0$, the second derivative is given by $\mathrm{d}^2 Z/\mathrm{d}R^2 = (\partial^2 Z/\partial S^2)(\mathrm{d}S/\mathrm{d}R)^2 > 0$,

where the inequality follows from the convexity of Z in S (by Lemma 4). Thus, any interior extremum of Z is a minimum so that Z is quasi-convex in R.

Since Z is quasi-convex in R it can only be maximized by either $R = 0$ or $R = \overline{R}$. Let $\Omega(R)$ be the equilibrium value of Z when the self-imposed rigidities take the value R. Then, $R = \overline{R}$ is optimal if and only if $\Omega(\overline{R}) - \Omega(0) \geq 0$ and $R = 0$ is optimal otherwise.

To show the first part of the proposition, namely that there exists a unique $\widehat{\phi}(p)$ such that $R = \overline{R}$ is optimal if $\phi \geq \widehat{\phi}(a, p)$ and $R = 0$ is optimal otherwise, we only need to show that if $\Omega(\overline{R}) - \Omega(0) = 0$, then $\Omega(\overline{R}) - \Omega(0)$ is increasing in ϕ. To see that this is indeed the case note first that

$$\left. \frac{\mathrm{d}(\Omega(\overline{R}) - \Omega(0))}{\mathrm{d}\phi} \right|_{\Omega(\overline{R}) - \Omega(0) = 0} = \frac{\partial \Omega(\overline{R})}{\partial S} \frac{\mathrm{d}S(\overline{R})}{\mathrm{d}\phi} - \frac{\partial \Omega(0)}{\partial S} \frac{\mathrm{d}S(0)}{\mathrm{d}\phi}. \tag{A-10}$$

Note next that because $\Omega(R)$ is quasi-convex in R, it must be the case that if $\Omega(\overline{R}) - \Omega(0) = 0$, then

$$\frac{\mathrm{d}\Omega(\overline{R})}{\mathrm{d}R} = \frac{\partial \Omega(\overline{R})}{\partial S} \frac{\mathrm{d}S(\overline{R})}{\mathrm{d}R} > 0 \quad \text{and} \quad \frac{\mathrm{d}\Omega(0)}{\mathrm{d}R} = \frac{\partial \Omega(0)}{\partial S} \frac{\mathrm{d}S(0)}{\mathrm{d}R} < 0.$$

Since, as shown above, $\mathrm{d}S/\mathrm{d}R < 0$ for all R, it follows that $\partial \Omega(\overline{R})/\partial S < 0$ and $\partial \Omega(0)/\partial S > 0$ if $\Omega(\overline{R}) - \Omega(0) = 0$. This, together with the observation that $\mathrm{d}S/\mathrm{d}\phi < 0$ for all R, implies that the expression in (A-10) is positive, which proves our claim.

Finally, to see that $\widehat{\phi}(a, p)$ is increasing in a we only need to show that if $\Omega(\overline{R}) - \Omega(0) = 0$, then $\Omega(\overline{R}) - \Omega(0)$ is decreasing in a. To see that this is indeed the case note that

$$\left. \frac{\mathrm{d}(\Omega(\overline{R}) - \Omega(0))}{\mathrm{d}a} \right|_{\Omega(\overline{R}) - \Omega(0) = 0} = \frac{\partial \Omega(\overline{R})}{\partial S} \frac{\mathrm{d}S(\overline{R})}{\mathrm{d}a} - \frac{\partial \Omega(0)}{\partial S} \frac{\mathrm{d}S(0)}{\mathrm{d}a}. \tag{A-11}$$

We have just shown that if $(\Omega(\overline{R}) - \Phi(0)) = 0$, then $\partial \Omega(\overline{R})/\partial S < 0$ and $\partial \Omega(0)/\partial S > 0$. This, together with the observation that $\mathrm{d}S/\mathrm{d}a > 0$ for all R, implies that the expression

in (A-11) is negative if $(\Omega(\overline{R}) - \Omega(0)) = 0$. The comparative statics for p can be shown in a similar manner. ∎

B Free Entry

In this section we consider a variant of the model in Section 3 with free entry by firms. Since the matching function displays constant returns to scale, we can write the probability with which a vacancy is matched with a worker as $am(u,v)/v = am(u/v,1) = aq(\theta)$, where $\theta \equiv v/u$ is a measure of labor market tightness and $q(\theta) \equiv m(1/\theta,1)$. The properties of the matching function imply that $q'(\theta) < 0$. Analogously, we have that the probability with which an unemployed worker is matched with an open vacancy is $am(u,v)/u = a\theta q(\theta) \equiv a\chi(\theta)$ and that $\chi'(\theta) > 0$. We impose the normalization $a = 1$ from now on.

The analysis in Sections 4.1 and 4.2 are unaffected by free entry. The market equilibrium analyzed in Section 4.3 however does change. In particular, we now have

$$(1-\delta)U = b + \chi(\theta)\delta\left[W_0 - U\right] \tag{A-12}$$

and

$$(1-\delta)V = -c + q(\theta)\delta\left[J_0 - V\right]. \tag{A-13}$$

Free entry ensure that $V = 0$ so that (A-13) can be restated as

$$c = q(\theta)\delta J_0. \tag{A-14}$$

We can use (A-12) and (A-14) together with (1), (2), (7) and (15) to prove that Proposition 1 continues to hold.

Proposition A1 (Equilibrium with Free Entry): *There exists a unique free entry equilibrium.*

Proof: To prove this proposition we will show that (A-12) implies a positive relationship between S and θ and (A-14) implies a negative relationship. Together these two equations can therefore be used to solve for the unique equilibrium values of S and θ. Note that by $S = U + V - \phi - R$ and $V = 0$, this maps into a unique equilibrium value for U. Here, we normalize $R = 0$ so as to simplify the expressions.

Using (1), (3), $S_0 = S + \phi$ and $V = 0$ to totally differentiate (A-12) gives

$$\left[1 - \delta + \frac{\chi(\theta)\delta}{2}\left(1 - \frac{\mathrm{d}Z}{\mathrm{d}S}\right)\right]\mathrm{d}S + \left[1 - \delta + \frac{\chi(\theta)\delta}{2}\right]\mathrm{d}\phi = \frac{\chi'(\theta)\delta}{2}[Z - S - \phi]\mathrm{d}\theta. \quad \text{(A-15)}$$

Together with $\mathrm{d}\phi = 0$, this implies that S is increasing in θ when (A-12) holds:

$$\frac{\mathrm{d}S}{\mathrm{d}\theta} = \frac{\delta(Z - S - \phi)\chi'(\theta)}{2(1 - \delta) + \chi(\theta)\delta(1 - \mathrm{d}Z/\mathrm{d}S)} > 0,$$

where the inequality follows from $\chi'(\theta) > 0$ and $\mathrm{d}Z/\mathrm{d}S < 1$ as shown in the proof of Lemma 1. Similarly, using (2), (3), $S_0 = S + \phi$ and $V = 0$ to totally differentiate (A-14) gives

$$q(\theta)\left(1 - \frac{\mathrm{d}Z}{\mathrm{d}S}\right)\mathrm{d}S + q(\theta)\mathrm{d}\phi = q'(\theta)[Z - S - \phi]\mathrm{d}\theta. \quad \text{(A-16)}$$

Together with $\mathrm{d}\phi = 0$, this implies that S is increasing in θ when (A-14) holds:

$$\frac{\mathrm{d}S}{\mathrm{d}\theta} = \frac{q'(\theta)[Z - S - \phi]}{q(\theta)(1 - \mathrm{d}Z/\mathrm{d}S)} < 0,$$

where the inequality follows from $q'(\theta) < 0$ and $\mathrm{d}Z/\mathrm{d}S < 1$. ∎

We can also use the above equations to proof that Lemma 2 continues to hold.

Lemma A2: *With free entry, a reduction in the firing tax ϕ increases the expected joint value of separation for a firm-worker pair S, i.e. $\mathrm{d}S/\mathrm{d}\phi < 0$.*

Proof: Using (A-15) and (A-16) to eliminate $\mathrm{d}\theta$ and dividing each term by $\mathrm{d}\phi$ gives

$$\frac{\mathrm{d}S}{\mathrm{d}\phi} = -\frac{2(1 - \delta) + \delta B(\theta)}{2(1 - \delta) + \delta B(\theta)(1 - \mathrm{d}Z/\mathrm{d}S)} < 0, \quad \text{(A-17)}$$

43

where

$$B(\theta) \equiv \mu(\theta) + \frac{\chi'(\theta)q(\theta)}{-q'(\theta)} > 0. \qquad \text{(A-18)}$$

This completes the proof. ∎

Using $V = 0$, $S_0 = S + \phi$ and (3), we have $\mathrm{d}U = \mathrm{d}S + \mathrm{d}\phi$ and, using (A-17), we obtain

$$\frac{\mathrm{d}U}{\mathrm{d}\phi} = \frac{\delta B(\theta)(-\mathrm{d}Z/\mathrm{d}S)}{2(1 - \delta) + \delta B(\theta)(1 - \mathrm{d}Z/\mathrm{d}S)}.$$

The denominator is always positive and the sign of the numerator is equal to sign $(-\mathrm{d}Z/\mathrm{d}S)$. We therefore have the intuitive result that an increase in ϕ makes an unemployed worker better off if and only if the match value Z is decreasing in S (and thus increasing in ϕ).

The proof of Lemma 3 in the text is unaffected by free entry. This allows us to establish the comparative statics stated in the next proposition.

Proposition A2(Comparative Statics with Free Entry): *With free entry,*

(a) the effect of a marginal change in ϕ on Z satisfies:.

$$\frac{\mathrm{d}Z}{\mathrm{d}\phi} = \begin{cases} \geq 0 & \text{if } \phi > \overline{\phi} \\ < 0 & \text{if } \phi < \underline{\phi}. \end{cases}$$

(b) the effect of a marginal change in ϕ on $S_0 = U + V$ satisfies

$$\frac{\mathrm{d}S_0}{\mathrm{d}\phi} = \frac{\mathrm{d}U}{\mathrm{d}\phi} = \begin{cases} \geq 0 & \text{if } \phi > \overline{\phi} \\ < 0 & \text{if } \phi < \underline{\phi}. \end{cases}$$

Proof: Since ϕ only affects Z through S we have from (7) and (15) that

$$\frac{\mathrm{d}Z}{\mathrm{d}\phi} = \frac{\mathrm{d}Z}{\mathrm{d}S}\left(\frac{\mathrm{d}U}{\mathrm{d}\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} - 1\right) = \frac{\mathrm{d}Z}{\mathrm{d}S}\left(\frac{\mathrm{d}U}{\mathrm{d}\phi} - 1\right),$$

where the second equality follows from $\mathrm{d}V/\mathrm{d}\phi = 0$ due to free entry. In the proof of Lemma A2 we have shown that $\mathrm{d}U/\mathrm{d}\phi < 1$. Thus, sign $(\mathrm{d}Z/\mathrm{d}\phi) = -\text{sign}\ (\mathrm{d}Z/\mathrm{d}S)$. Part (a) then follows from Lemma 3.

For part (b) note that because of free entry $dS_0/d\phi = dU/d\phi$. In the proof of Lemma A2 we have shown that $\text{sign}\,(dU/d\phi) = -\text{sign}\,(dZ/dS)$. Part (b) then follows from Lemma 3. ∎

Finally, we can state the following proposition.

Proposition A3 (Welfare with Free Entry): *Define aggregate welfare as the per-capita (average) payoff. Then, with free entry, a small reduction in the firing tax reduces welfare when labor market frictions are high, i.e. when $\phi \geq \overline{\phi}$.*

Proof: Social welfare is given by $uS_0/2 + (1-u)Z/2$. From Proposition A2 we know that S_0 and Z are increasing in ϕ when $\phi \geq \overline{\phi}$. Since $Z \geq S_0$ we only need to show that the unemployment rate u is decreasing in ϕ when $\phi \geq \overline{\phi}$. Following the reasoning in the proof of Lemma A1, the unemployment rate is given by

$$u = \left[1 + \frac{2\chi(\theta)}{2\delta + \lambda\left\{E_\rho[1 - G(\widetilde{\gamma})] + E_\gamma[F(\widetilde{\rho})]\right\}}\right]^{-1}. \tag{A-19}$$

The firing tax affects the unemployment rate through two channels. First, as in the model without free entry, it affects unemployment through the probability of disagreement $E_\rho[1 - G(\widetilde{\gamma})] + E_\gamma[F(\widetilde{\rho})]$. In particular, for the same reasons as in the model without free entry, an increase in ϕ reduces the probability of disagreement which, in turn, decreases the unemployment rate. Second, in the model with free entry the firing tax also affects market tightness and, through this channel, the probability $\chi(\theta)$ with which unemployed workers find vacancies and thus leave unemployment. As we will show next, when $\phi \geq \overline{\phi}$ an increase in the firing tax increases θ and thus increases the probability $\chi(\theta)$ of exiting unemployment, thereby lowering the unemployment rate u. For this purpose, we use (A-15), (A-16) and (A-18) to obtain

$$\frac{d\theta}{d\phi} = 2(1-\delta)q\,(\theta)\,\frac{-dZ/dS}{-q'(\theta)\,[2(1-\delta) + \delta B(\theta)(1 - dZ/dS)]}.$$

The denominator is strictly positive so that $\text{sign}\,(\mathrm{d}\theta/\mathrm{d}\phi) = \text{sign}\,(-\mathrm{d}Z/\mathrm{d}S)$. It then follows from Lemma 3 that $\mathrm{d}\theta/\mathrm{d}\phi \geq 0$ when $\phi \geq \overline{\phi}(a)$. $\blacksquare$

C Efficient Bargaining

In this section we analyze the model described in Section 6. Most of the analysis in Section 4.1. still goes through. In particular, W_0 is still given by (1) and J_0 is still given by (2). The expected utility W_1 of a worker who formed an employment relationship in the past and has not yet been hit by the shock is given by (4) and the corresponding expected value of a firm J_1 is given by (5). Since we are assuming that the shock only hits once, we must now also specify the expected utility of an employed worker after a shock has hit, W_2, and the value of a firm with a filled vacancy after a shock has hit, J_2. These values are given by respectively by (4) and (5) for $\lambda = 0$, in which case $Z = p/(1 - \delta)$ by (7).

Next, we need to reconsider the analysis in Section 4.2. To describe the bargaining solution of the bargaining game that takes place when a firm and a worker have been hit by a shock, we can apply the Revelation Principle which allows us to restrict the analysis, without loss of generality, to Bayesian incentive compatible direct mechanisms. Suppose then that, after having learned ρ and γ, the firm and the worker make announcements $\widehat{\rho}$ and $\widehat{\gamma}$. A direct mechanism specifies the probability of trade $q(\widehat{\gamma}, \widehat{\rho})$ and expected adjustment bonus $\beta(\widehat{\gamma}, \widehat{\rho})$ as a function of these announcements. The bargaining game maximizes the match value subject to the interim participation constraints. Thus, it is described by the solution to

$$\max_{q(\cdot),\,\beta(\cdot)} Z^e = S + E_{\gamma, p}\left[(\rho - \gamma + W_2 + J_2 - S)\, q(\gamma, \rho)\right] \tag{A-20}$$

subject to the interim participation and incentive compatibility constraints, where $Z_2 \equiv$

$(W_2 + J_2)$.[19] Note that Z_2 and S do not depend on γ and ρ and can therefore be treated as constants. The following lemma follows immediately from Myerson and Satterthwaite (1983) and describes the optimal trading rule that solves the maximization problem (A-20).

Lemma A3: *The optimal trading rule that solves maximization problem (A-20) is given by*

$$q(\gamma, \rho) = \begin{cases} 1 & \text{if } p - \gamma + Z_2 - S \geq \frac{\mu}{1+\mu} \left(H(\gamma) + K(\rho) \right) \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu \in [0, \infty)$. For $S - Z_2 \leq \inf P - \sup \Gamma$, $\mu = 0$ and for $S - Z_2 > \inf P - \sup \Gamma$, $\mu > 0$ solves the interim participation constraint

$$E_{\gamma,\rho} \left[(\rho - \gamma + Z_2 - S - K(\rho) - H(\gamma)) \, q(\gamma, \rho) \right] = 0. \tag{IR}$$

Proof: This lemma follows immediately from Myerson and Satterthwaite (1983) and we refer to their analysis. ■

Thus, even if the firm and the worker play the most efficient bargaining game, bargaining is inefficient if $S - Z_2 > \inf P - \sup \Gamma$. Essentially, as long as continuation of the employment relationship is not optimal for all ρ and γ, the firm and the worker sometimes separate although it would be efficient for them to continue the employment relationship.

Having described the bargaining game at the renegotiation stage, we can proceed and close the model by solving for U and V as in Section 4.3. The equations (16) - (18) still hold and we can use them to show the existence and uniqueness of the equilibrium of the model.

---

[19]The incentive compatibility constraints ensure that each manager finds it optimal to make truthful announcements of his or her type and the interim individual rationality constraints ensure that, after learning their type, the agents prefer participating in the bargaining game to realizing the disagreement payoffs.

Proposition A4 (Equilibrium): *The equilibrium of the model exists and is unique.*

Proof: The proof is similar to that in Proposition 1. We only need to show that with the different bargaining that we now assume it is still the case that $\mathrm{d}Z/\mathrm{d}S < 1$ as shown in Lemma 1. We can use Lemma A3 and (7) and apply the envelope theorem to get

$$\frac{\mathrm{d}Z}{\mathrm{d}S_0} = \frac{\delta\lambda}{1 - \delta(1 - \lambda)} \left\{ (1 - \mathrm{E}_{\gamma,p}[q(\gamma,\rho)]) - \mu \mathrm{E}_{\gamma,p}[q(\gamma,\rho)] \right\} \leq 1. \tag{A-21}$$

For the rest of the proof we refer to the proof of Proposition 1. ■

Next we need to reconsider the analysis in Sections 4.4. Lemma 2 still goes through since its proof does not depend on the assumed bargaining game; indeed, the key step in that lemma requires $\mathrm{d}Z/\mathrm{d}S$ to be smaller than unity, which holds by (A-21). Thus, it is still the case that a reduction in the firing tax increases S and therefore makes it less costly for firms and workers to separate. The effect of such a reduction on Z is also the same as in the main model. In particular, on the one hand, a reduction in the firing tax makes it less costly for firms and workers to separate, which tends to increase Z, but on the other hand, it also increases the probability of disagreement, which tends to reduce Z. To see this formally consider (A-21). The first term in the squared brackets represents the positive effect of a marginal increase in S: the firm and the worker realize a higher payoff whenever they disagree, which happens with probability $(1 - E_{\gamma,\rho}[q(\gamma,\rho)])$. The second term represents the costs of a marginal increase in S: the probability of agreement is reduced since the participation constraint is more binding.

In contrast to the main model, we can no longer characterize analytically when either of the two effects dominates for general distributions. In other words, we can no longer prove something as general as Lemma 3. We can, however, show that for a number of common distributions our key results still hold.

48

Lemma A4: *Suppose that the distributions of γ and ρ are either the standard normal, standard exponential, or standard uniform distributions. Then Z is quasi-convex in S, has an interior minimum, and reaches its global maximum as S goes towards its upper limit.*

Proof: This follows from Proposition 3 and Footnote 17 in Matouschek (2004). ∎

For these common distributions all the results in Sections 4.4 and 5 then continue to hold. In particular, the proofs of Propositions 2 - 5 still go through.

Finally, we turn to the numerical example described in Section 6. Note first that for this example $Z_2 = 1$. It then follows from Lemma A3 that a firm-worker pair that is hit by a shock continues its relationship with probability

$$q(\gamma, \rho) = \begin{cases} 1 & \text{if } p - \gamma \geq \frac{1+\mu}{1+2\mu}(S_0 - \phi - 1) + \frac{\mu}{1+2\mu} \\ 0 & \text{otherwise,} \end{cases} \tag{A-22}$$

where $\mu \in [0, \infty)$. For $(S_0 - \phi - 1) \leq -1$, $\mu = 0$ and for $(S_0 - \phi - 1) > -1$, $\mu > 0$ solves the interim participation constraint

$$E_{\gamma, \rho}\left[(2(\rho - \gamma) + S_0 - \phi - 2)q(\gamma, \rho)\right] = 0. \tag{A-23}$$

Substituting $Z^e = \{S + E_{\gamma, p}[(\rho - \gamma + 1 - S)q(\gamma, \rho)]\}$ into (7) gives

$$Z = \frac{p + \lambda\delta\{S + E_{\gamma, p}[(\rho - \gamma + 1 - S)q(\gamma, \rho)]\}}{1 - \delta(1 - \lambda)}. \tag{A-24}$$

We can use equations (18) and (A-22) - (A-24) to solve for μ, S_0 and Z. Equations (1) - (5) can then be used to solve for W_0, J_0, W_1 and J_1. Finally, equations (16) and (17) give U and V.
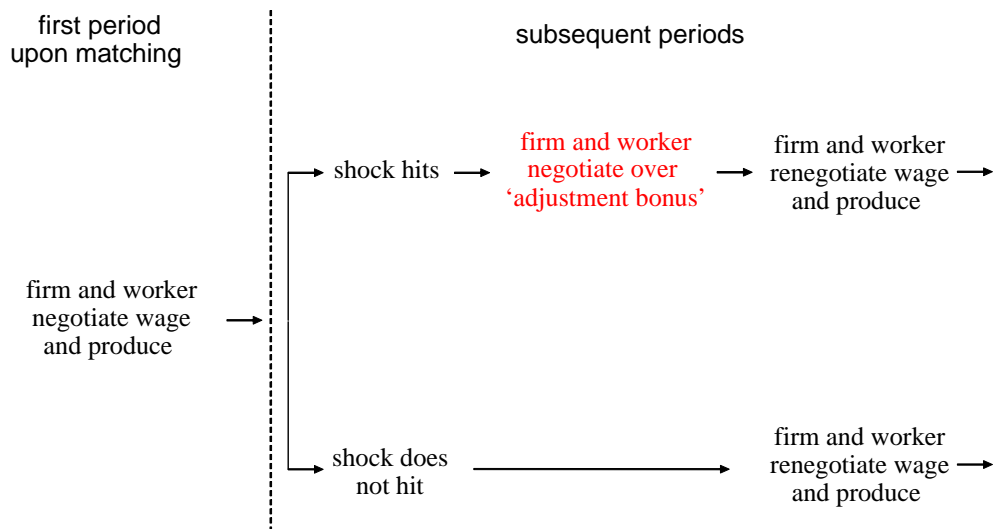
Figure 1: Timing

References

Alonso-Borrego, C., J. Fernández-Villaverdeand J. Galdon-Sanchez, "Evaluating Labor Market Reforms: A General Equilibrium Approach," IZA Discussion Papers 1129 (2004), Institute for the Study of Labor (IZA).

Alvarez, F. and M. Veracierto, "Severance Payments in an Economy with Frictions," *Journal of Monetary Economics*, 47 (2001), 477-98.

Ausubel, L. P. Cramton and R. Deneckere, "Bargaining with Incomplete Information," in the *Handbook of Game Theory* (R. Aumann and S. Hart, eds.), Vol. 3 (2002), Amsterdam: Elsevier Science B.V.

Bergemann, A. and A. Mertens, "Job Stability Trends, Layoffs and Quits- An Empirical Analysis for West-Germany," mimeo, Humboldt University, 2001.

Burguet, R., R. Caminal, and C. Matutes, "Golden Cages for Showy Birds; Optimal Switching Cost in Labor Contracts," *European Economic Review* 46 (2002), 1153-1185.

Copic, J. and C. Ponsati,, "Ex-Post Efficiency of Mechanisms for Bilateral Trade with Risk-Averse Traders," mimeo, California Institute of Technology and Institut D'Analisi Economica 2004.

Dore, R., "The End of Jobs for Life? Corporate Employment Systems: Japan and Elsewhere," Occasional Paper 11, Centre for Economic Performance, London School of Economics, 1996.

Faig, M. and B. Jerez, "A Theory of Commerce," *Journal of Economic Theory* 122 (2005), 60-99.

Fella, G., "Efficiency Wage and Efficient Redundancy Pay," European Economic Review 44 (2000), 1473-1490.

Friedberg, L. and A. Webb, "Retirement and the Evolution of Pension Structure," NBER Working Paper 9999 (2003), forthcoming in Journal of Human Resources.

Garibaldi, P. and E. Wasmer, "Labor Market Flows and Equilibrium Search Unemployment," *Journal of the European Economic Association* 3 (2005), 851-882.

Givord, P. and E. Maurin, "Changes in Job Security and their Causes: An Empirical Analysis for France, 1982-2002," *European Economic Review* 48 (2004), 595-615.

Güell, M., "Fixed-term Contracts and Unemployment: An Efficiency Wage Analysis," mimeo, Pompeu Fabra, 2003.

Guerrieri, V., "Efficiency of Competitive Search under Asymmetric Information", mimeo (2005), MIT.

Hall, R., "Lost Jobs," *Brookings Papers on Economic Activity* 1 (1995), 221-256.

Hall, R. and E. Lazear, "The Excess Sensitivity of Layoffs and Quits to Demand," *Journal of Labor Economics* 2 (1984), 233-257.

Kagel, J. and A. Roth *Handbook of Experimental Economics* (Princeton: Princeton University Press, 1997).

Kranton, R., "Reciprocal Exchange: A Self-Sustaining System," *American Economic Review*, 86(1996), 830-851.

Matouschek, N., "Ex Post Inefficiencies in a Property Rights Theory of the Firm," *Journal of Law, Economics, and Organization* 20 (2004), 125-147.

Matouschek, N. and P. Ramezzana, "The Role of Exclusive Contracts in Facilitating Market Transactions,", *Journal of Industrial Economics*, forthcoming (2007).

Matouschek, N., P. Ramezzana and F. Robert-Nicoud, "Labor Market Frictions, Job Instability, and the Flexibilityof the Employment Relationship," CEPR Discussion Paper 4193 (2005).

McLaren, J. and **A. Newman**, "Globalization and Insecurity," mimeo, University of

Virginia and University College London, 2002.

Moen, E. and A. Rosen, "Incentives in Competitive Search Equilibrium and Wage Rigidity," mimeo (2005), Norwegian School of Management.

Myerson, R. and **M. Satterthwaite**, "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory* 28 (1983), 265-281.

Neumark, D. (editor), *On the Job : Is Long-Term Employment a Thing of the Past?* (New York: Russel-Sage Foundation, 2000).

Pissarides, C., *Equilibrium Unemployment Theory*, 2nd edition (Cambridge, Mass.: MIT Press, 2000).

Pissarides, C., "Employment Protection," Labour Economics 8 (2001), 131-159.

Ramey, G. and J. Watson, "Bilateral Trade and Opportunism in a Matching Market," *Contributions to Theoretical Economics* 1 (2001).

Saint-Paul, G., "Efficiency Wage, Commitment and Hysteresis," Annales d'Economie et de Statistique 37–38 (1995), 39–53.

Saint-Paul, G., *Dual Labor Markets* (Cambridge, Mass.: MIT Press, 1996).

Saint-Paul, G., *The Political Economy of Labor Market Institutions* (Oxford: Oxford University Press, 2000).

Scheve, K. and M. Slaughter, "Economic Insecurity and the Globalization of Production," *American Journal of Political Science* 48 (2004), 662-674.

Schmitz, P., "Job Protection Laws and Agency Problems under Asymmetric Information," *European Economic Review* 48 (2004), 1027-1046.

Segal, L. and D. Sullivan, "The Growth of Temporary Services Work," *Journal of Economic Perspectives* 11 (1997), 117-136.

Shimer, R. and R. Wright, "Competitive Search Equilibrium with Asymmetric Information," mimeo (2004), University of Chicago.

Thesmar, D. and M. Thoenig, "Why Is a Flexible World More Insecure? The Way Outsourcing Amplifies Uncertainty," mimeo, INSEE and CERAS, 2002.

Williams, S., "A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms," *Economic Theory*, 14 (1999), 155-180.