The Novelty of Innovation: Competition, Disruption, and Antitrust Policy[*]

Steven Callander[†]   Niko Matouschek[‡]

October 13, 2020

Abstract

Innovation is not just a matter of doing things better. It is also a matter of doing things differently. We develop a model to capture the novelty of innovation and explore what it means for the nature of market competition and quality of innovations. We show that a variant of the famed Arrow replacement effect holds in that new entrants pursue more innovative technologies than do incumbents. This result is reversed, however, when the incumbent can acquire the entrant post-innovation. The prospect of acquisition makes innovation more profitable but simultaneously suppresses the novelty of innovation as the entrant seeks to maximize her value to the incumbent. This reversal suggests a positive role for a strict antitrust policy that spurs entrepreneurial firms to innovate boldly. The model also yields new insight into the nature of competition in innovative industries, when disruption occurs, and why firms acquire competitors only to shut them down.

1 Introduction

Innovation is not just a matter of doing things better. It is also a matter of doing things differently. Sometimes these differences are small, sometimes they are enormous, with ideas and technologies that depart radically from those that came before.

This presents budding entrepreneurs with an additional choice. They must decide *how* to innovate and not just whether to innovate. What technology should they pursue in their efforts at change? The more radical an innovation is, the more likely they achieve a radical breakthrough, but also the more likely they suffer a radical failure.

The choice of technology with which to innovate also matters for the nature of market competition. The further a technology is from the mainstream—from an industry's dominant design—the more different is the set of consumers it serves and, therefore, the softer is the competition with the incumbent firm.

The market entry strategy of a new firm must balance these dual considerations of technology and market competition. In this paper we explore and characterize the incentives to innovate in such an environment, focusing on the novelty of the innovations undertaken. This allows us to see not only whether there is innovation in a market, but how novel it is, and what that means for market efficiency, both in terms of the technological improvements it delivers and in the degree of market competition it generates. We then trace through the implications for firm strategy in terms of mergers and acquisitions. Our main result is to show that a lax antitrust policy—in which incumbents can merge with entrepreneurial upstarts—has a chilling effect on innovation. While the prospect of a takeover may enhance an entrant's incentive to innovate, we show that it pushes the entrant towards less novel innovations, so much so that the entrant can be less innovative than the incumbent firm would be on its own without the pressures of competition.

To get at these issues, we introduce a novel model of technological uncertainty in which the possibilities for innovation are rich and fall along a continuum, as also do the outcomes. This covers the full gamut of innovation, from incremental steps to bold innovations, from breakthrough outcomes to memorable flops, and everything in between.[1] In the model, a new market entrant chooses how radical they wish to be. A critical feature of the modeling technology is that the uncertainty over the outcome of the innovation increases in the radicalness of the experiment. This captures the idea that the more we move from what is known—the greater the distance from existing technology—the more uncertain is the outcome. At the same time, the more novel is the innovation, the more distant it is from the mainstream taste of consumers and the lower is the intensity of market competition with the incumbent firm.[2]

---

[1] Our definition of innovation follows that of Rogers (1962): "An idea, practice, or object that is perceived as new by an individual or other unit of adoptions." It does not require that the outcome be a success.

[2] This is in the spirit of Christensen (1997) that disruptive innovations first appear in niches of existing markets. Christensen argues this is typically at the bottom end of the market although, infamously, the iPhone entered at the top end.

We begin our analysis with a classic question: Is innovation more likely to come from an independent entrepreneur or an established firm? We differ from classic accounts in that we examine the question not in terms of the amount, or intensity, of innovation but in terms of the novelty of innovation. Nevertheless, we show that an amended "Arrow replacement effect" continues to hold in that the incumbent always innovates less radically and more incrementally than does the entrepreneur (Arrow, 1962). We refer to this as the "spatial Arrow effect" as it relates to the differences between innovations rather than the intensity of innovation.[3]

Despite the similarity, the logic for our result differs from Arrow's. Arrow's argument is that the incumbent has less incentive to innovate as it already has the dominant product and, so, only receives the marginal gain from an innovation. In our setting, this logic pushes the incumbent's innovation *away* from its existing product. The more novel is the new product, the less it cannibalizes the existing product, and the more the incumbent's product portfolio appeals to the broad range of customer tastes. In contrast, as Arrow pointed out, the independent entrepreneur has the added incentive of stealing market share from the incumbent. The classic Arrovian logic suggests, therefore, that the entrepreneur will stay tight to the incumbent as by doing so it can potentially gain more of the market.

To see why the entrepreneur nevertheless differentiates more, we must draw in another classic intuition, that of Hotelling's spatial competition. Because a more novel innovation is more distant also in terms of market competition, the entrepreneur has the incentive to differentiate to soften market competition. The incumbent, being able to coordinate pricing across products, does not have this incentive. We show that the competition-softening incentive of the entrepreneur dominates the market spanning incentive of the incumbent, and the spatial Arrow effect follows. In this way, the combination of classic insights from spatial differentiation and price competition yield new insights into classic questions of innovation.

These differences lead to subtle but important implications about what it means to disrupt. Disruption depends not only on the entrant's technology. It also depends on how distant the entrant is in the technology space, for consumers may still prefer an incumbent product if the new technology, even if better, does not serve their needs as well. We show that this leads to an inversion of the standard logic of who disrupts and when it happens. We show that while the independent entrepreneur engages in a bolder experiment, and thus has a higher probability of a breakthrough outcome, she disrupts the incumbent product with smaller probability than does the incumbent disrupt itself. That is to say, the probability that the incumbent product remains in the market is lower when the incumbent itself innovates and innovates more incrementally. Self-disruption in this way is less dramatic—the innovation may only be a marginal improvement—yet given its proximity in the technology space, it is enough to render the incumbent product obsolete. In contrast, the entrepreneur may possess a markedly better product yet still not drive the incumbent from the

---

[3]One might also think of this as the *extensive* versus the *intensive* margin of innovation.

market. This pattern goes some way to explaining why apparently disruptive innovations don't necessarily drive incumbent firms from the market, with many firms lingering for years.[4]

We then use the model to explore deeper questions about firm strategy and antitrust policy in innovative industries. These questions have risen to prominence in recent years in academic debate and, even more so, in the public domain. The seeming increase in market concentration in the U.S. and a corresponding decline in investment has led many to question whether accommodative antitrust policy has led to the suppression of innovation.[5]

We investigate these questions by allowing the incumbent firm to take over, or merge, with the entrepreneurial upstart. Such a move is anti-competitive and, thus, desirable by the incumbent. The question of our interest, then, is not whether the merger occurs, but what effect its prospect has on the *ex ante* incentives of an entrepreneur to innovate and the novelty of the innovation she chooses.

We show that the prospect of being acquired by the incumbent causes the entrepreneur to moderate in her innovation, choosing a more incremental innovation that is closer in technological space to the incumbent. In fact, we show that this effect is sufficiently strong that it overturns the spatial Arrow effect when the entrepreneur has enough bargaining power. In this case the prospect of merging induces the entrepreneur to choose a more incremental innovation than would the incumbent itself.

The spatial Arrow effect itself suggests that the entrant would want to distance herself from the incumbent as a different technology that spans more of the market would be more valuable to the incumbent, imitating what the incumbent would do itself. That much is true. But a second force drives in the opposite direction. By moving toward the incumbent, the entrepreneur poses a greater competitive threat and, thus, ensures that her removal is of even greater value to the incumbent. This pushes the entrepreneur toward rent seeking behavior, to act against efficiency because doing so improves her bargaining leverage over the incumbent. We show that this force dominates, overturning the spatial Arrow effect and undermining innovation in the market.

In Silicon Valley, as elsewhere in the world, the goal of many entrepreneurs is not an IPO, rather it is to be acquired by an existing incumbent firm, such as Google, Microsoft, and the like. This practice is often pointed to as evidence that innovation is flourishing. Even though the market power of the large tech firms may appear overwhelming, the argument goes, acquisitions provide a viable and valuable channel for innovation. Our result shows the dark side of this practice. A lax antitrust policy that allows dominant firms to gobble up emerging competitors certainly does provide a spur to innovation, but it does so in way that pushes entrepreneurs toward incremental innovations. Thus, the market, and society at large, miss out on the bold and breakthrough innovations that

---

[4]Lepore (2014) offers the examples of Seagate Technology, US Steel, and Bucyrus in a prominent critique of Christensen (1997).

[5]See Philippon (2019) for a book length treatment with particular emphasis on the diverging paths of the U.S. and the E.U.

otherwise would emerge if competition were guaranteed.

A merger delivers to the incumbent the dual benefits of spanning the market and reducing competition. This latter incentive is sufficiently strong that, in the extreme, the incumbent will buy the entrant with the express intention of shutting it down, what Cunningham et al. (2020) have colorfully labelled "killer acquisitions." The logic for killer acquisitions in our model is distinct from that of Cunningham et al. (2020) and, in contrast to their result, we find that killer acquisitions occur for a broader range of outcomes the more different is the entrant's innovation from the incumbent's product. The insight of Cunningham et al. (2020) is that development costs are more duplicative when technologies are more similar. We complement their explanation by showing that market-competition effects alone are sufficient to justify a killer acquisition, and that this is more easily satisfied the more distant are the technologies.

Killer acquisitions occur when the entrepreneur's innovation fails to meet expectations but only moderately so. When this occurs, the innovation delivers what we refer to as a "nuisance product." It is a nuisance in that its lower quality adds no market-spanning advantage to the incumbent and, if the incumbent controlled the technology, it would shelve it and shut down the firm. Nevertheless, when controlled by the independent entrepreneur, the innovation is sufficiently strong to coexist in the market with the incumbent, engaging in competition and damaging the incumbent's profit. This combination of factors is more easily satisfied the softer is price competition. Thus, the range of outcomes in which the entrant is a nuisance—and the incumbent desires a killer acquisition—the more distant is its innovation in the technology space.

Relationship to the Literature: A distinction between types of innovations has not gone unnoticed in the literature. Christensen (1997) famously articulated a typology of "sustaining" and "disruptive" innovations.[6] In Christensen's telling, however, the distinction is binary and market disruption is possible only with disruptive innovations and not with sustaining innovations. Our model shows that the distinction between sustaining and disruptive can be seen as one of degree rather than type, and that not only can an incumbent product be disrupted by an incremental innovation, it is, in fact, more likely to be so, even if is less dramatic than disruption that comes from bolder innovations.[7] Our logic is also distinct from Christensen as to why an incumbent firm experiments less boldly. According to Christensen, the incumbent fails to innovate because it is too focused on its existing customers. In our model, in contrast, the incumbent is not blinkered in this way and actually prefers a more distinct technology so as to span the market. Nevertheless, it is the entrepreneur who innovates more boldly as she must additionally account for price competition in the post-innovation market.

---

[6]This typology builds, in turn, on the even-more classic exploration vs. exploitation trade-off and applies it across firms.

[7]Fitzgerald et al. (2020) recently provided evidence that incremental innovations are under-appreciated by the market relative to bolder innovations.

Our approach also relates to work on firm competition in innovative industries. The choice of innovation and the richness of the alternative space is again the key distinction from that literature. The standard approach is to assume an exogenous innovation process and study how that affects the structure and, in particular, the dynamics of market competition.[8] Adner and Zemsky (2005) set up two distinct market segments and address the important question of whether and when two technologies compete. They use their model to explore the dynamics of this question, building upon the inductive reasoning of Christensen. In modeling competition on a Hotelling line, we presume competition (unless one product is dominated) and focus on how that competition shapes the incentives to innovate, thereby generating the process of innovation endogenously.[9]

In working back from post-innovation market behavior to the incentives to innovate, we follow a long line of work in economics and management. Gans and Stern (2000) combine these threads to explore a variety of competitive practices in the context of knowledge creation and what it means for the supply of R&D. We complement this and the rest of the literature by adding the novelty dimension to this question. Combining insights, it is easy to see how the novelty of innovation can be suppressed at the same time as the quantity of innovation increases, pointing to the need for more subtle interpretations of the efficiency of many competitive practices.

2 A Model of Innovation Novelty

We study competition between two firms, an incumbent and an entrant. The firms locate in the technology space, which is given by the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Consumers have heterogeneous preferences over the products these technologies produce, with their ideal points distributed uniformly over the space.

The incumbent firm makes product 0 which is located at the center of the market, denoted $l_0 = 0$, with known quality v_0. The entrant makes product 1 and enters the market by choosing a technology with which to produce this product. Without loss of generality, we set this to be $l_1 \in \left[0, \frac{1}{2}\right]$. Any product other than 0 is new and, thus, has unknown quality. We suppose that this uncertainty increases in the novelty of the innovation, which is measured by its distance from the technology underlying product 0. Specifically, after the entrant chooses l_1, v_1 is drawn from a Normal distribution with mean $v_0 + \mu l_1$ and variance $\sigma^2 l_1$. The variance parameter $\sigma^2 > 0$ captures the notion that there is more uncertainty about the quality of a new product, the further it is located from an existing one. The parameter $\mu \in [0, t]$, in turn, captures the extent to which

---

[8]An exception is Cabral (2003) that allows firms to choose the variance of their innovation, although the choice is limited to be binary (high or low). The focus of the analysis is then how that choice changes over the dynamic course of market competition. The model does not include horizontal differentiation and price competition that is central to our results.

[9]Meagher and Zauner (2004) extend the Hotelling model to allow for uncertainty over demand. In their model, uncertainty is exogenous, common across firms, and independent of the firms' strategies, which is very different from uncertainty due to innovation.

differentiation increases the new product's expected quality.[10] We denote the quality differential between the products as: $\Delta \equiv v_1 - v_0$. For simplicity, we set the marginal cost of production to zero for both firms.

The entrant incurs development costs $c(l_1) \geq 0$ for its new product. These costs are larger, the further the new product is from the existing one, reflecting the fact that it takes time and resources to research into the unknown. In particular, $c(0) = c'(0) = 0$ and $c'(l_1) \geq 0$ for all $l_1 \in \left[0, \frac{1}{2}\right]$. Furthermore, we assume $c(l_1)$ is sufficiently convex for the entrant's location problem to be well-behaved, and we make this precise in the appendix.

Each consumer, s, buys a single unit of a product. The utility from product $j \in \{0, 1\}$ at price p_j, decreases quadratically in the distance of that product from the consumer's ideal, as in the classic Hotelling (1929) model of spatial competition. Specifically,

$$u_{sj} = v_j - t(s - l_j)^2 - p_j,$$

where $t > 0$ represents the disutility of technology distance.[11] The reservation utility of buying neither product is zero. To ensure that all consumers buy one of the products, as is standard in Hotelling models, we assume that the quality of the incumbent's product satisfies:[12]

$$v_0 \geq \frac{5}{4}t. \tag{1}$$

We are interested in how ownership of the entrant affects innovation and market outcomes. To this end, we compare three cases: (i.) the entrant is independent, (ii.) the incumbent owns the entrant, and (iii.) the entrant is independent initially but can be acquired by the incumbent after its product has been developed and before prices are set. In the last case, the acquisition price is determined by Nash bargaining, where the entrant's bargaining power is given by $\alpha \in [0, 1]$ and the incumbent's by $(1 - \alpha)$.

At the beginning of the game, the entrant decides on the location of its product after which the product's quality is realized. Depending on the case we examine, the incumbent may then be able to acquire the entrant. Next, the firms simultaneously set prices $p_0 \in [0, \infty)$ and $p_1 \in [0, \infty)$ and do so non-cooperatively if the entrant is independent. Finally, consumers make their purchase decisions, profits are realized, and the game ends.

We characterize the unique subgame perfect equilibrium for all permutations of the model.

---

[10]The microfoundation for this formulation is that there exists a mapping from technology/product space to outcomes, and that this mapping is the realized path of a Brownian motion anchored at $(0, v_0)$, with drift μ and variance σ^2 (see Callander and Matouschek (2019) for more elaboration). We require $\mu \leq t$ as otherwise there is no trade-off between technology and consumers.

[11]Adner and Zemsky (2005) offer the interpretation of the Hotelling line as representing technological distance.

[12]This implies the market is 'covered.' In addition to tractability, this assumption is the best case for the Arrow replacement effect as all sales of the incumbent must cannibalize the existing product, adding robustness to our our reversal result (Proposition 4).

The derivation proceeds by backward induction, and we follow this logic in presenting the results, beginning with price setting competition between the two firms (or within the incumbent firm) once qualities are determined.

3 Prices, Market Shares, & Profits

As a benchmark, consider the incumbent's problem if the new product did not exist. In this case, the incumbent would charge the highest price at which all consumers are willing to buy the existing product,

$$\overline{p}_0 = v_0 - \frac{1}{4}t.$$

A higher price would cause the incumbent to lose marginal consumers on the flank, and the condition in (1), along with the uniform distribution of consumers, implies this is not profitable. Charging a lower price would not win any new customers and would simply turn profits into consumer surplus. The price $\overline{p}_0$ generates profit $\overline{\pi}_0$, and since there is a unit mass of consumers and zero marginal cost of production, we have $\overline{\pi}_0 = \overline{p}_0$. In what follows, we refer to $\overline{p}_0$ and $\overline{\pi}_0$ as the "status quo" price and profits.

Suppose now the new product does exist and is either owned by the independent entrant or the incumbent. Figure 1 illustrates equilibrium prices and the implied market share for the new product for the two cases with fixed location $l_1 > 0$, where the superscripts C and M stand for competition and monopoly (see Lemmas A1 & A2 in the appendix for the corresponding expressions).

Begin with the case in which the incumbent owns the new product. As the figure indicates, the incumbent continues to charge the status quo price $\overline{p}_0$ for the existing product as this appeals to the consumers on the left flank and ensures they all make a purchase. As Arrow pointed out, sales of the new product necessarily come at the expense of the existing product. To be profitable, therefore, the price charged for the new product must be at least as high as for the existing product. As consumers are closer to the entrant technologically, the entrant can capture market share from these consumers even if its quality is lower. The quality differential cannot be too large, however, for this to be the case. Specifically, the new product fails to capture any market share if the quality differential, $\Delta \equiv v_1 - v_0$, satisfies

$$\Delta \leq -t l_1 \left(1 - l_1\right). \tag{2}$$

Similarly, the existing product will not be able to hold on to any market share if the quality of the new product is sufficiently high that

$$\Delta \geq t l_1 \left(3 + l_1\right). \tag{3}$$

In between these two thresholds, the incumbent sells both products and it sells more of the new product, at a higher price, the larger the quality differential is. Proposition 1 describes the profit to the incumbent from this pricing strategy.
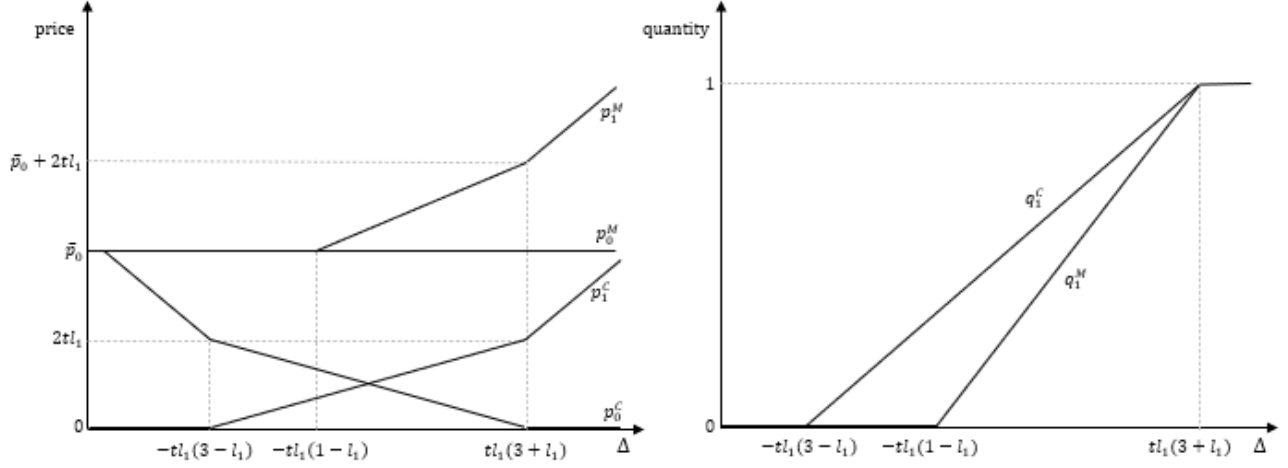
Figure 1: Equilibrium Prices and Market Shares for fixed $l_1 > 0$ and Varying Quality

Proposition 1 *If the incumbent owns the entrant, its profits are*

$$\pi_0^M + \pi_1^M = \begin{cases} \overline{\pi}_0 + \Delta - tl_1\left(1 + l_1\right) & if \;\; \Delta \geq tl_1\left(3 + l_1\right) \\ \overline{\pi}_0 + \frac{1}{8tl_1}\left(\Delta + tl_1\left(1 - l_1\right)\right)^2 & if \;\; -tl_1\left(1 - l_1\right) \leq \Delta \leq tl_1\left(3 + l_1\right) \\ \overline{\pi}_0 & if \;\; \Delta \leq -tl_1\left(1 - l_1\right). \end{cases}$$

When the entrant is independent, the two firms compete over price within the market. As a result, prices fall and the new product captures a larger share of the market. The market share and price of the new product are again increasing in the quality differential Δ and the new product still takes over the entire market if the differential satisfies (3). The lower threshold below which the new product fails to capture any market share, though, is lower. Specifically, under competition the new product fails to capture any market share if

$$\Delta \leq -tl_1\left(3 - l_1\right), \tag{4}$$

as illustrated in Figure 1.

If the quality differential is between the two lower thresholds, that is, if $-tl_1\left(3 - l_1\right) \leq \Delta \leq -tl_1\left(1 - l_1\right)$, the new entry is what we call a nuisance product. Its quality is high enough to steal market share from the incumbent when the firms are competing but not high enough to attract any consumers at the high price the incumbent would want to sell it at in the absence of any competitive pressure.

Proposition 2 describes the profit from this competition for the two firms.

9

Proposition 2 *If the entrant is independent, the incumbent's profits are*

$$
\pi_0^C = \begin{cases}
0 & \text{if } \Delta \geq tl_1 (3 + l_1) \\
\frac{1}{18tl_1} (\Delta - tl_1 (3 + l_1))^2 & \text{if } -tl_1 (3 - l_1) \leq \Delta \leq tl_1 (3 + l_1) \\
-\Delta - tl_1 (1 - l_1) & \text{if } \frac{1}{4}t - tl_1 (1 - l_1) - v_0 \leq \Delta \leq -tl_1 (3 - l_1) \\
\overline{\pi}_0 & \text{if } \Delta \leq -tl_1 (1 - l_1) - \left(v_0 - \frac{1}{4}t\right)
\end{cases}
$$

and the entrant's profits are

$$
\pi_1^C = \begin{cases}
\Delta - tl_1 (1 + l_1) & \text{if } \Delta \geq tl_1 (3 + l_1) \\
\frac{1}{18tl_1} (\Delta + tl_1 (3 - l_1))^2 & \text{if } -tl_1 (3 - l_1) \leq \Delta \leq tl_1 (3 + l_1) \\
0 & \text{if } \Delta \leq -tl_1 (3 - l_1).
\end{cases}
$$

Notice that, in both propositions, profits are convex in the quality differential Δ when the market share of each product is strictly positive. This reflects our previous observation that, for this case, both the price and market share of the new good are increasing in Δ and those of the existing product are decreasing. As a result, the firms are risk loving with respect to Δ (despite being risk neutral over profit itself), providing them with an incentive to take on risk by differentiating the new product from the existing one.

4   Innovation

At the beginning of the game, the innovator chooses the location of the new product. If the incumbent owns the entrant, the optimal location is

$$
l_1^M = \max_{l_1 \in \left[0, \frac{1}{2}\right]} \mathrm{E}\left[\pi_0^M + \pi_1^M\right] - c(l_1), \tag{5}
$$

where $c(l_1)$ are the development costs. If, instead, the entrant is independent, the optimal location is

$$
l_1^C = \max_{l_1 \in \left[0, \frac{1}{2}\right]} \mathrm{E}\left[\pi_1^C\right] - c(l_1). \tag{6}
$$

The assumption that $c(l_1)$ is sufficiently convex for the objective functions to be strictly concave ensures that the optimal locations are unique. We can now establish the spatial Arrow effect.

Proposition 3 *The new product is more differentiated from the existing one if the entrant is independent than if it is owned by the incumbent, that is,*

$$
l_1^C > l_1^M.
$$

10

To see why $l_1^C > l_1^M$, begin by noting that by differentiating and rearranging the expected profit expressions in Propositions 1 and 2, we obtain

$$\frac{\mathrm{d}\pi_1^C}{\mathrm{d}l_1} \geq \frac{\mathrm{d}\left(\pi_0^M + \pi_1^M\right)}{\mathrm{d}l_1} \tag{7}$$

for any fixed quality differential $\Delta \in \mathbb{R}$ and $l_1 \in \left[0, \frac{1}{2}\right]$. The impact of a marginal increase in location l_1 on π_1^C and $\pi_0^M + \pi_1^M$ is the same if either $\Delta \leq -tl_1\left(3 - l_1\right)$, in which case nobody is buying the new product, or $\Delta \geq tl_1\left(3 + l_1\right)$, in which case everybody is buying it. Otherwise, the impact is always strictly larger when the entrant is independent than when it is owned by the incumbent.

In this intermediate range, dueling forces are pulling the firms in opposite directions. The classic intuition from Arrow is that the independent entrant has more market share by locating closer to the incumbent. In contrast, the incumbent benefits from diversification, from pushing its new product further away. To see this, recall that the incumbent always sells the new product at a higher price than the existing one. If the consumers who prefer the new product are all to the right, then the incumbent strictly prefers locating its new product closer to them as this allows a higher price to be charged. Working in the opposite direction is a second classic intuition, that from price competition. The independent entrant benefits from locating further away as this softens price competition with the incumbent, increasing both market share and the price it can charge. The incumbent, who can coordinate prices across products, does not possess this incentive. The relationship in (7) establishes that it is the latter effect that dominates—the price competition effect—and that it is the independent entrant who benefits from differentiation in the technology space.

The differentiation in (7) holds quality constant, such that, for any given distribution of quality, the impact of a marginal increase in l_1 on expected profits $\mathrm{E}\left[\pi_1^C\right]$ is larger than that on $\mathrm{E}\left[\pi_0^M + \pi_1^M\right]$. As the variance in outcomes varies in l_1 itself, the final step in the proof of Proposition 3 is to show that the inequality still holds once we take this into account. That is, it shows

$$\frac{\mathrm{d}\mathrm{E}\left[\pi_1^C\right]}{\mathrm{d}l_1} > \frac{\mathrm{d}\mathrm{E}\left[\pi_0^M + \pi_1^M\right]}{\mathrm{d}l_1}$$

for all $l_1 \in \left[0, \frac{1}{2}\right]$. This then implies the result $l_1^C > l_1^M$.

From this result, we can explore the probability that the new innovation disrupts the market and drives the incumbent product out. The inequality in Proposition 3 implies that the quality of the entrant's innovation is more uncertain, and, thus, more likely to be very high or very low. This is easy to see from the Normal distribution, as depicted in Figure 2. Let z be a standard normal random variable and let $\mathrm{F}\left(z\right)$ denote its cumulative density function. Then, for any fixed threshold
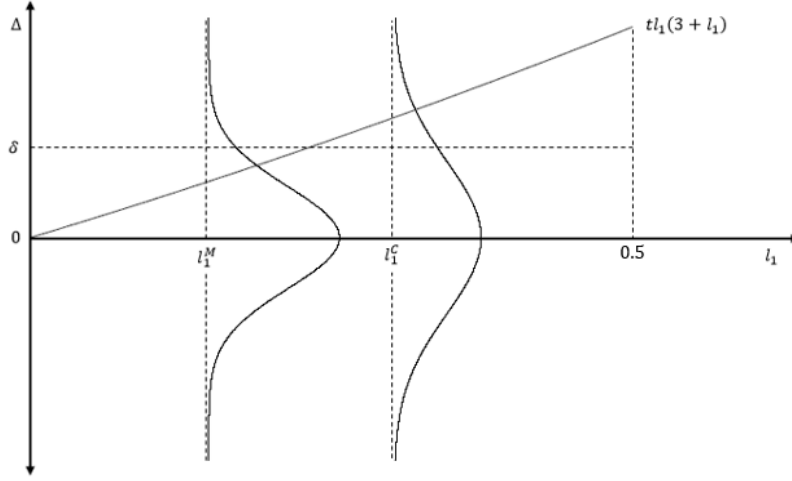
Figure 2: Disruption Probability; $\mu = 0$

$\delta > 0$,

$$\text{prob}\left(\Delta \geq \delta\right) = \text{prob}\left(z \geq \frac{1}{\sigma\sqrt{l_1}}\left(\delta - \mu l_1\right)\right) = 1 - F\left(\frac{1}{\sigma\sqrt{l_1}}\left(\delta - \mu l_1\right)\right)$$

and

$$\frac{\text{dprob}\left(\Delta \geq \delta\right)}{\text{d}l_1} = \frac{\delta + \mu l_1}{2\sigma l_1\sqrt{l_1}}f\left(\frac{1}{\sigma\sqrt{l_1}}\left(\delta - \mu l_1\right)\right) > 0.$$

This property accords with our intuitions about bolder innovations having more chance of a break-through (and, conversely, more chance of being a flop). It does not imply, however, that the entrant has more chance of disrupting the market. Recall from (3) that the existing product has zero market share and is wiped out if $\Delta \geq t l_1(3 + l_1)$, regardless of whether the entrant is independent or not. Critically, this requirement is quadratic in l_1, as depicted in Figure 2.

The probability of disruption, then, is the probability that the quality of the entrant exceeds this increasing threshold. As the novelty of the innovation increases, this becomes a race between the threshold and increasing variance. Formally, the probability of disruption is

$$\text{prob}\left(q_0^j = 0\right) = 1 - F\left(\frac{1}{\sigma\sqrt{l_1}}\left(t l_1(3 + l_1) - \mu\right)\right) \text{ for } j = C, M.$$

Differentiating shows that the race is won by the threshold as the probability of disruption is decreasing in l_1:

$$\frac{\text{dprob}\left(q_0^j = 0\right)}{\text{d}l_1} = -\frac{\mu + 3t l_1\left(l_1 + 1\right)}{2\sigma l_1\sqrt{l_1}}f\left(\frac{1}{\sigma\sqrt{l_1}}\left(\delta - \mu l_1\right)\right) < 0.$$

From Proposition 3, we then have the following.

Corollary 1 *The existing product's market share is zero with higher probability when the entrant is incumbent-owned than when it is independent.*

It is worth noting that a similar procedure establishes that the entrepreneur is less likely to completely fail—to disappear from the market—than is the incumbent-owned entrant. The entrepreneur has a higher chance of a bad outcome, but in a reflection of the threshold for disruption, the threshold for complete failure is decreasing and concave, and this effect dominates the increased variance. This property may shed light on why incumbent firms are so often thought to be bad at innovation and to engage in so many failures. As the model illustrates, incumbents rationally innovate incrementally and this choice sets a high bar for success, a higher bar than for independent entrants who have more competitive freedom precisely because they innovate more boldly.

5   Acquisitions & Antitrust

Suppose now the entrant owns the new product but can be acquired by the incumbent. Since the firms must be jointly better off coordinating prices, and there are no diseconomies of scale, the firms' joint gains from merging are always positive, that is,

$$\pi_0^M + \pi_1^M - \left(\pi_0^C + \pi_1^C\right) \geq 0.$$

Moreover, since there are no informational asymmetries or other bargaining frictions, the parties always agree on a merger.

The firms Nash bargain over the gains, where the entrant's bargaining power is given by $\alpha \in [0, 1]$ and the incumbent's by $1 - \alpha$. If the incumbent and the entrant do not agree on a merger, they compete for customers and realize profits π_0^C and π_1^C. Given these outside options, and the quality of the new product, the incumbent's expected profits are then

$$\pi_0^C + (1 - \alpha) \left(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C\right)$$

and the entrant's are

$$\pi_1^C + \alpha \left(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C\right).$$

Even though the parties always agree to merge, the motive and implications of the merger depend on the location and quality of the new product, as is summarized in Figure 3. The key thresholds in this calculus include the cut-points in market share expressed earlier in (2), (3), and (4), and reflected in Figure 1, as well as one new threshold.

The new threshold, given in (8) is the quality of the entrant's innovation below which she not only fails to gain any market share, but fails to restrain the incumbent's pricing power in any way. This reflects an innovation that substantially underperforms expectations; it is the lowest threshold

in Figure 3. This occurs when no consumer is willing to buy the new product even if it is given away for free and the existing product is sold at the status quo price $\overline{p}_0$. That is, if

$$\Delta \leq -tl_1 (1 - l_1) - \overline{p}_0. \tag{8}$$

The gains to a merger in this case is zero. The firms are not hurt by merging, but they also don't benefit from it, and the entrant receives a price of zero upon merging.

For a quality differential that is somewhat higher, in particular, if

$$-tl_1 (1 - l_1) - \overline{p}_0 < \Delta \leq -tl_1 (3 - l_1),$$

the new product still cannot capture any market share even if the entrant gives it away for free (the right-hand side is (4)). Nevertheless, a zero price on the entrant's product in this range constrains what price the incumbent can charge for the existing price, and it is forced to lower the price below the status quo. The new product, in other words, *contests* the market for the existing one despite not gaining any market share. Acquiring the entrant doesn't increase the incumbent's market share but it does allow it to remove this impediment to high monopoly prices. The merger is valuable, therefore, and the entrant receives a positive price despite it having no profitable future as an independent company.

Suppose next that the quality differential is good enough to capture market share in a competitive industry but not good enough to attract consumers at the high price the monopolist would sell it at. That is, if the quality differential is between the thresholds given by (2) and (4),

$$-tl_1 (3 - l_1) < \Delta \leq -tl_1 (1 - l_1).$$

As we saw above, the new entrant is then a nuisance product. (This can be seen in the right-side panel in Figure 1). In this case, the incumbent acquires the entrant to shut it down. In the terminology of Cunningham et al. (2020), the incumbent engages in a killer acquisition. This possibility emerges if and only if the innovation fails to meet expectations, but only slightly so. The range when it is possible, as indicated in Figure 3 and explained in the introduction, expands with the novelty of the innovation.

Acquiring the new product to kill it no longer makes sense for the incumbent if the quality differential is larger. In particular, if

$$-tl_1 (1 - l_1) < \Delta < tl_1 (3 + l_1),$$

the new product is sufficiently valuable that some consumers are willing to buy it at more than the status quo price (the differential Δ is between the thresholds in (2) and (3)). This is the standard case for a market power merger as it allows the firm to charge a higher price for both products
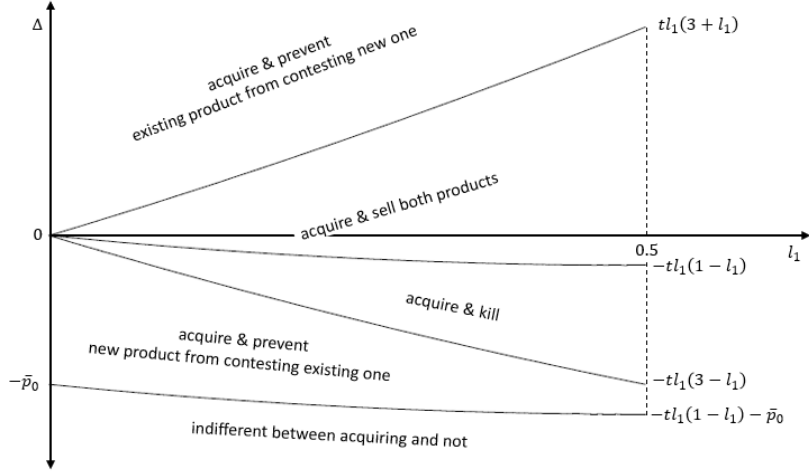
Figure 3: Acquisition Strategy of the Incumbent Firm

without eliminating either from the market. The new product is valuable even if it is the same quality as it enables the incumbent to serve more of the market with products tailored to separate segments.

The final case is when the quality of the entrant's innovation is so high that it dominates the market and the incumbent product is disrupted. This is exactly (3):

$$\Delta \geq tl_1 \left(3 + l_1\right).$$

Although the incumbent is disrupted, the merger is valuable as the market is still contested, although this time it is the market for the new product that is contested rather than the market for the existing product. The incumbent receives a positive price regardless of the quality of the new product as its product remains valuable to consumers for a sufficiently low but still positive price. This holds for arbitrarily high quality of the new product and there is no symmetric case to when the entrant's low quality renders it irrelevant.[13]

With the acquisition strategy of the firms in hand, we can now, by backward induction, determine the impact mergers have on the incentive to innovate, in particular on the novelty of the innovation a new entrant undertakes.

As we did in Section 4, the entrepreneur chooses a new technology to maximize her profitability. In this case she does so with post-innovation *and* post-merger outcomes in mind. Her optimal choice

---

[13]We are tempted to refer to this as a "seppuku acquisition," in which the incumbent acquires the new firm only to kill itself, although formally, because $v_0 > 0$, the existing product is only shut down when it would have zero market share even in a competitive market.

15

is now given by:

$$l_1^A(\alpha) = \max_{l_1 \in [0, \frac{1}{2}]} \mathrm{E}\left[\pi_1^C + \alpha\left(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C\right)\right] - c(l_1)$$

where the superscript A stands for acquired. We can now establish our main result.

Proposition 4 *An independent entrant that is later acquired has an optimal location $l_1^A(\alpha)$ that is decreasing in its bargaining power and satisfies $l_1^A(0) = l_1^C$ and $l_1^A(1) \in \left(0, l_1^M\right).$*

This says that the spatial Arrow effect is reversed if the entrant has sufficient bargaining power. The independent entrant innovates more incrementally and with less novelty than would the incumbent firm if it controlled the entrant from the start. This effect increases in the entrant's bargaining power.

For the characterization of the choice l_1^A, notice that we can rewrite the entrant's expected profits as

$$\mathrm{E}\left[\pi_1^C + \alpha\left(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C\right)\right] = \mathrm{E}\left[(1 - \alpha)\pi_1^C + \alpha\left(\pi_0^M + \pi_1^M\right) - \alpha\pi_0^C\right].$$

If the entrant has no bargaining power, $\alpha = 0$, the entrant expects to make the profits $\mathrm{E}\left[\pi_1^C\right]$, and thus chooses the same location $l_1^A(0) = l_1^C$, as when it cannot be acquired. If, instead, the entrant has all the bargaining power, its expected profits are $\mathrm{E}\left[\pi_0^M + \pi_1^M - \pi_0^C\right]$, which is less than the expected profits, $\mathrm{E}\left[\pi_0^M + \pi_1^M\right]$, the incumbent expects to make when it owns the entrant from the start. Now notice that $\mathrm{E}\left[\pi_0^C\right]$ is increasing in l_1: the more the new product is differentiated from the existing one, the higher incumbent's expected profits if it ends up competing with the independent entrant. It then follows that an entrant who knows it will be acquired by the incumbent chooses a location to the left of the one chosen by the incumbent-owned entrant, $l_1^A(1) < l_1^M$. The reason is that locating closer to the existing product lowers the incumbent's outside option, the profits it can expect to make if it does not acquire the entrant. This lower outside option, in turn, forces it to pay a higher prices for the entrant.

We saw earlier that an independent entrant (that cannot be acquired) disrupts the existing product less frequently than does the incumbent-owned entrant disrupt itself. The same logic holds here, although now, when the Arrow effect is reversed, it is the entrant who has a higher chance of disruption. However, we should not laud such entrepreneurs. For despite disrupting the incumbent, they are able to do so only because they innovate so tepidly, varying the existing product only incrementally. It is precisely because they locate so close to the existing product that only a small quality premium is needed to disrupt. The rent-seeking behavior of such entrepreneurs may be more likely to lead to market turnover—and it certainly increases their own pay-off—but it lowers overall innovation and market efficiency, and we are all the lesser for it.

6 Conclusion

The question of antitrust policy in innovative industries has recently received considerable attention, both in academic research and in the popular domain. The issues at the heart of this debate match the object of our analysis. Is a market more innovative when controlled by an incumbent with market power? Or is innovation better served by creating space for entrepreneurial upstarts and market competition? These questions of antitrust reach a climax when policymakers must decide whether to allow the incumbent to takeover the upstart. Should Facebook be allowed to buy Instagram? Amazon buy Zappos or AT&T T-Mobile? In expanding analysis to include the novelty of innovation, our model provides an additional dimension by which policymakers can understand more deeply the market and the incentives that antitrust policy provides.

This analysis recasts and complements the existing focus of the antitrust debate. Typically, the focus in analyzing a merger is on the implications for future innovations. Our model emphasizes instead the ex ante impact of antitrust policy. For instance, it implies that if investors anticipate lax antitrust enforcement, they will fund entrepreneurs with incremental ambitions and eschew those pursuing bold change. Our hope is that the model we develop provides a way to understand these ex ante incentives and the impact they have on the innovativeness of a market.

In expanding the domain of analysis for innovation and antitrust policy, we set aside other important dimensions, and there is much that our model leaves out. One prominent feature we leave out is market competition dynamics. Many researchers emphasize the dynamics of disruption that come from new innovations (Christensen, 1997; Adner and Zemsky, 2005). In modeling only a single period of market competition, our model cannot speak to this feature, and extending our model of innovation novelty in this direction offers substantial promise. Nevertheless, the static analysis provides more than just a starting point as it illuminates what role dynamics actually play in disruption.[14] We show that disruption can emerge in a single period and can come from the lower (or upper) end of the market, as Christensen suggests it typically does, even if it is immediate in the model whereas it takes some time in practice.

References

Adner, R. and P. Zemsky (2005, February). Disruptive technologies and the emergence of competition. *The RAND Journal of Economics 36*(2), 229–254.

Arrow, K. (1962, April). *The Rate and Direction of Inventive Activity: Economic and Social Factors.* Princeton University Press.

---

[14]See Gans (2011) for an argument that static analyses are often sufficient for, and at least more predictive of, market behavior in many environments.

Cabral, L. M. B. (2003, February). R&d competition when firms choose variance. *Journal of Economics and Management Strategy 12*(1), 139–150.

Callander, S. and N. Matouschek (2019, February). The risk of failure: Trial and error learning and long-run performance. *American Economic Journal: Microeconomics 11*(1), 44–78.

Christensen, C. M. (1997, June). *The innovator's dilemma: when new technologies cause great firms to fail.* Harvard Business School Press.

Cunningham, C., F. Ederer, and S. Ma (2020, forthcoming). Killer acquisitions. *Journal of Political Economy*.

Fitzgerald, T., B. Balsmeier, L. Fleming, and G. Manso (2020, September). Innovation search strategy and predictable returns. *Management Science*, 1–29.

Gans, J. S. and S. Stern (2000, Winter). Incumbency and r&d incentives: Licensing the gale of creative destruction. *Journal of Economics and Management Strategy 9*(4), 485–511.

Hotelling, H. (1929, March). Stability in competition. *Economic Journal 39*, 41–57.

Lepore, J. (2014, June). The disruption machine: What the gospel of innovation gets wrong. *The New Yorker*.

Meagher, K. J. and K. G. Zauner (2004). Product differentiation and location decisions under demand uncertainty. *Journal of Economic Theory 117*, 201–216.

Philippon, T. (2019, October). *The Great Reversal: How America Gave up on Free Markets.* Harvard University Press.

Rogers, E. (1962, August). *Diffusion of Innovations.* Free Press of Glencoe.

7 Appendix

Lemma 1 *If the incumbent owns the entrant, it charges $p_0^M = \bar{p}_0$ for the existing product and*

$$
p_1^M = \begin{cases}
\bar{p}_0 + \Delta - tl_1 (1 + l_1) & \text{if } \Delta \geq tl_1 (3 + l_1) \\
\bar{p}_0 + \frac{1}{2} (\Delta + tl_1 (1 - l_1)) & \text{if } - tl_1 (1 - l_1) \leq \Delta \leq tl_1 (3 + l_1) \\
\bar{p}_0 & \text{if } \Delta \leq -tl_1 (1 - l_1)
\end{cases}
$$

for the new one. The market share of the new product is given by

$$
q_1^M = \begin{cases}
1 & \text{if } \Delta \geq tl_1 (3 + l_1) \\
\frac{1}{2} + \frac{1}{4tl_t} (\Delta - tl_1 (1 + l_1)) & \text{if } - tl_1 (1 - l_1) \leq \Delta \leq tl_1 (3 + l_1) \\
0 & \text{if } \Delta \leq -tl_1 (1 - l_1).
\end{cases}
$$

Proof: The consumer indifferent between consuming products 0 and 1 is located at

$$
x = \frac{1}{2} l_1 - \frac{1}{2tl_1} ((v_1 - p_1) - (v_0 - p_0)). \tag{9}
$$

The firm's profit maximization problem is then given by

$$
\max_{p_0,, p_1} p_0 \left(x + \frac{1}{2} \right) + p_1 \left(\frac{1}{2} - x \right).
$$

which, after substituting x gives, $p_0 = \bar{p}_0$,

$$
p_1 = \left(\frac{1}{4} (2 (v_1 + v_0) - t + 2tl_1 (1 - l_1)) \right),
$$

and

$$
x = \frac{1}{4tl_1} (v_0 - v_1 + tl_1 (l_1 + 1)).
$$

This results in three different regimes: i. $x \geq 1/2$ when $\Delta \geq tl_1(3 + l_1)$, where the entrant captures the entire market, ii. $x \in (-1/2, 1/2)$ when $\Delta \in (-tl_1(1 - l_1), tl_1(3 + l_1))$, where both incumbent and entrant have positive market shares, and iii. $x \leq -1/2$ when $\Delta \leq -tl_1(1 - l_1)$, where the incumbent captures the entire market. ∎

Lemma 2 *If the entrant is independent, the incumbent charges*

$$
p_0^C = \begin{cases}
0 & \text{if } \Delta \geq tl_1 (3 + l_1) \\
\frac{1}{3} (-\Delta + tl_1 (3 + l_1)) & \text{if } - tl_1 (3 - l_1) \leq \Delta \leq tl_1 (3 + l_1) \\
-\Delta - tl_1 (1 - l_1) & \text{if } \frac{1}{4} t - tl_1 (1 - l_1) - v_0 \leq \Delta \leq -tl_1 (3 - l_1) \\
\bar{p}_0 & \text{if } \Delta \leq -tl_1 (1 - l_1) - \left(v_0 - \frac{1}{4} t \right)
\end{cases}
$$

19

and the entrant charges

$$p_1^C = \begin{cases} \Delta - tl_1\left(1 + l_1\right) & \text{if } \Delta \geq tl_1\left(3 + l_1\right) \\ \frac{1}{3}\left(\Delta + tl_1\left(3 - l_1\right)\right) & \text{if } -tl_1\left(3 - l_1\right) \leq \Delta \leq tl_1\left(3 + l_1\right) \\ 0 & \text{if } \Delta \leq -tl_1\left(3 - l_1\right). \end{cases}$$

The market share of the new product is given by

$$q_1^C = \begin{cases} 1 & \text{if } \Delta \geq tl_1\left(3 + l_1\right) \\ \frac{1}{2} + \frac{1}{6tl_1}\left(\Delta - tl_1^2\right) & \text{if } -tl_1\left(3 - l_1\right) \leq \Delta \leq tl_1\left(3 + l_1\right) \\ 0 & \text{if } \Delta \leq -tl_1\left(3 - l_1\right). \end{cases}$$

Proof: Given the location (9) of the consumer who is indifferent between the two products, the incumbent's and problem is given by

$$\max_{p_0} p_0 \left(\frac{1}{2} + \frac{1}{2tl_1}\left(v_0 - v_1 + p_1 - p_0 + l_1^2 t\right)\right)$$

and the entrant's problem is given by

$$\max_{p_1} p_1 \left(\frac{1}{2} - \frac{1}{2tl_1}\left(v_0 - v_1 - p_0 + p_1 + l_1^2 t\right)\right).$$

This results in

$$p_0 = \frac{1}{3}\left(v_0 - v_1 + tl_1\left(3 + l_1\right)\right),$$

$$p_1 = \frac{1}{3}\left(v_1 - v_0 + tl_1\left(3 - l_1\right)\right),$$

and

$$x = \frac{1}{2} - \frac{1}{6tl_1}\left(v_1 - v_0 + tl_1\left(3 - l_1\right)\right).$$

We again have three regions: i. $x \geq 1/2$ when $\Delta \geq tl_1(3 + l_1)$, where the entrant captures the entire market, ii. $x \in (-1/2, 1/2)$ when $\Delta \in (-tl_1(3 - l_1), tl_1(3 + l_1))$, where both incumbent and entrant have positive market shares. and iii. $x \leq -1/2$ when $\Delta \leq -tl_1(3 - l_1)$, where the incumbent captures the entire market. Moreover, in the incumbent-only region, the incumbent is constrained by the consumer at $1/2$ if the consumer prefers the entrant's product at $p_1 = 0$ over the incumbent's product, which holds for the range $\frac{1}{4}t - tl_1(1 - l_1) - v_0 \leq \Delta \leq -tl_1(3 - l_1)$. ■

Proof of Propositions 1 and 2: Both follow from Lemmas 1 and 2. ■

Proof of Proposition 3: Given the profit expressions in Propositions 1 and 2, we calculate the

difference in expected profits which results in $\overline{\pi}_0$ and two integral terms across two of the v_1 regions:

$$\mathrm{E}\left(\pi_0^M + \pi_1^M\right) - \mathrm{E}\left(\pi_1^C\right) \tag{10}$$

$$= \overline{\pi}_0 - \int_{v_0 - l_1 t(3 - l_1)}^{(v_0 - l_1 t(1 - l_1))} \frac{1}{18} \frac{(v_1 + l_1 t(3 - l_1) - v_0)^2}{l_1 t} \mathrm{dG}(v_1)$$

$$+ \int_{(v_0 - l_1 t(1 - l_1))}^{(v_0 + l_1 t(l_1 + 3))} \frac{1}{8} \frac{(v_0 - l_1 t(1 - l_1) - v_1)^2}{l_1 t} - \frac{1}{18} \frac{(v_1 + l_1 t(3 - l_1) - v_0)^2}{l_1 t} \mathrm{dG}(v_1),$$

where $\mathrm{G}(v_1)$ is the CDF for $\mathrm{N}\left(v_0 + \mu l_1, l_1 \sigma^2\right)$.

Next we use change of variables $v_1 = v_0 + \mu l_1 + \sqrt{l_1}\sigma z$ to get the integrals in terms of standard normal distributed variable z, which simplifies to

$$\mathrm{E}\left(\pi_0^M + \pi_1^M - \overline{\pi}_0\right) - \mathrm{E}\left(\pi_1^C\right) \tag{11}$$

$$= -\frac{\sigma^2}{18t} \int_{-\frac{1}{\sigma}\sqrt{l_1}(t(3 - l_1) + \mu)}^{-\frac{1}{\sigma}\sqrt{l_1}(t(1 - l_1) + \mu)} \left(\frac{1}{\sigma}\sqrt{l_1}\left(t(3 - l_1) + \mu\right) + z\right)^2 \mathrm{dF}(z)$$

$$+ \frac{\sigma^2}{72t} \int_{-\frac{1}{\sigma}\sqrt{l_1}(t(1 - l_1) + \mu)}^{\frac{1}{\sigma}\sqrt{l_1}(t(l_1 + 3) - \mu)} \left(\begin{array}{c} 9\left(\frac{1}{\sigma}\sqrt{l_1}\left(t(1 - l_1) + \mu\right) + z\right)^2 \\ -4\left(\frac{1}{\sigma}\sqrt{l_1}\left(t(3 - l_1) + \mu\right) + z\right)^2 \end{array} \right) \mathrm{dF}(z).$$

We differentiate the expected profit difference using both the Fundamental Theorem of Calculus and Differentiation under the Integration Sign. All of the Fundamental Theorem of Calculus terms sum up to 0, leaving just the two terms from differentiation under the integration signs:

$$\frac{\mathrm{d}\left[\mathrm{E}\left(\pi_1^M + \pi_1^M - \overline{\pi}_0\right) - \mathrm{E}\left(\pi_1^C\right)\right]}{\mathrm{d}l_1} \tag{12}$$

$$= -\frac{\sigma^2}{18t} \frac{1}{\sigma\sqrt{l_1}} \left(3t(1 - l_1) + \mu\right) \int_{-\frac{1}{\sigma}\sqrt{l_1}(t(3 - l_1) + \mu)}^{-\frac{1}{\sigma}\sqrt{l_1}(t(1 - l_1) + \mu)} \left(\frac{1}{\sigma}\sqrt{l_1}\left(t(3 - l_1) + \mu\right) + z\right) \mathrm{dF}(z)$$

$$- \frac{\sigma}{72t} \frac{1}{\sqrt{l_1}} \int_{-\frac{1}{\sigma}\sqrt{l_1}(t(1 - l_1) + \mu)}^{\frac{1}{\sigma}\sqrt{l_1}(t(l_1 + 3) - \mu)} \left(\frac{1}{\sigma}\sqrt{l_1} \left(\begin{array}{c} 3t^2(5l_1 + 9)(1 - l_1) \\ +\mu(2t(10l_1 + 3) - 5\mu) \\ + (3t(5l_1 + 1) - 5\mu) z \end{array} \right) \right) \mathrm{dF}(z) \tag{13}$$

As both of these terms of negative, we get the differential inequality

$$\frac{\mathrm{d}\left[\mathrm{E}\left(\pi_1^C\right)\right]}{\mathrm{d}l_1} > \frac{\mathrm{d}\left[\mathrm{E}\left(\pi_1^M + \pi_1^M - \overline{\pi}_0\right)\right]}{\mathrm{d}l_1}. \tag{14}$$

The result that $l_1^C > l_1^M$ then follows from our assumption that the cost function $c(l_1)$ is sufficiently convex for the objective functions in (5) and (6) to be strictly concave. The final step is to show

that such a cost function exists. To this end we show that, for all $l_1 \in [0, 1/2]$,

$$\frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_1^C \right) \right]}{\mathrm{d}l_1^2} \quad \text{and} \quad \frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_1^M + \pi_1^M \right) \right]}{\mathrm{d}l_1^2}$$

are bounded from above. To this end, we proceed as we did above to find

$$
\begin{aligned}
\frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_1^C \right) \right]}{\mathrm{d}l_1^2} &= \frac{1}{4\sqrt{l_1}l_1} \left[\frac{1}{3\sigma} l_1 \left(3t - \mu + 3tl_1 \right)^2 f \left(-\frac{1}{\sigma\sqrt{l_1}} \left(\mu l_1 - tl_1 \left(l_1 + 3 \right) \right) \right) \right. \\
&\quad + \frac{1}{9t} \int_{-\frac{\sqrt{l_1}}{\sigma}(\mu+t(3-l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu-t(3+l_1))} \left(4t\sqrt{l_1}l_1 \left(-6t - 2\mu + 3tl_1 \right) - z\sigma \left(3tl_1 - \left(3t + \mu \right) \right) \right) \mathrm{dF}\left(z \right) \\
&\quad \left. - \int_{-\frac{\sqrt{l_1}}{\sigma}(\mu-t(3+l_1))}^{\infty} 8\sqrt{l_1}l_1 t + z\sigma \mathrm{dF}\left(z \right) \right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_0^M + \pi_1^M \right) \right]}{\mathrm{d}l_1^2} &= \frac{1}{4l_1\sqrt{l_1}} \left[\frac{1}{4t} \int_{-\frac{\sqrt{l_1}}{\sigma}(\mu+t(1-l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu-t(3+l_1))} 4tl_1\sqrt{l_1} \left(-2t - 2\mu + 3tl_1 \right) \mathrm{dF}\left(z \right) \right. \\
&\quad - \frac{1}{4t} \int_{-\frac{\sqrt{l_1}}{\sigma}(\mu+t(1-l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu-t(3+l_1))} z\sigma \left(t + \mu + 3tl_1 \right) \mathrm{dF}\left(z \right) \\
&\quad \left. - \int_{-\frac{\sqrt{l_1}}{\sigma}(\mu-t(3+l_1))}^{\infty} 8t\sqrt{l_1}l_1 + z\sigma \mathrm{dF}\left(z \right) \right].
\end{aligned}
$$

Notice that both expressions are continuous and finite for $l_1 = 1/2$. Since it is also the case that

$$\lim_{l_1 \to 0} \frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_1^C \right) \right]}{\mathrm{d}l_1^2} = \lim_{l_1 \to 0} \frac{\mathrm{d}^2 \left[\mathrm{E}\left(\pi_0^M + \pi_1^M \right) \right]}{\mathrm{d}l_1^2} = \lim_{l_1 \to 0} \frac{1}{4\sqrt{l_1}l_1} \left(-\int_0^\infty z\sigma \mathrm{dF}\left(z \right) \right) = -\infty$$

it then follows that both expressions are bounded from above for all $l_1 \in [0, 1/2]$. ∎

Proof of Proposition 4: As noted in the text, we can write the entrant's expected profits as

$$\mathrm{E}\left[\pi_1^C + \alpha \left(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C \right) \right] = \mathrm{E}\left[\left(1 - \alpha \right) \pi_1^C + \alpha \left(\pi_0^M + \pi_1^M \right) - \alpha \pi_0^C \right].$$

The proposition then follows from (14) and the fact that

$$\frac{\mathrm{d} \left[\mathrm{E}\left(\pi_0^C \right) \right]}{\mathrm{d}l_1} > 0. \tag{15}$$

22

To show that this inequality holds, we use the expression in Proposition 2 to obtain

$$
\mathrm{E}\left(\pi_0^C\right) = \int_{-\infty}^{v_0 - tl_1(1-l_1) - \left(v_0 - \frac{1}{4}t\right)} \overline{\pi}_0 \mathrm{dG}\left(v_1\right)
$$

$$
+ \int_{v_0 - tl_1(1-l_1) - \left(v_0 - \frac{1}{4}t\right)}^{v_0 - tl_1(3-l_1)} -\Delta - tl_1\left(1 - l_1\right) \mathrm{dG}\left(v_1\right)
$$

$$
+ \int_{v_0 - tl_1(3-l_1)}^{v_0 + tl_1(3+l_1)} \frac{1}{18tl_1}\left(\Delta - tl_1\left(3 + l_1\right)\right)^2 \mathrm{dG}\left(v_1\right).
$$

Next, we substitute $v_1 = v_0 + \mu l_1 + \sqrt{l_1}\sigma z$, differentiate with respect to l_1, and reverse the substitution using $z = \left(v_1 - v_0 - \mu l_1\right)/\sqrt{l_1}\sigma$ to obtain

$$
\frac{\mathrm{d}\left[\mathrm{E}\left(\pi_0^C\right)\right]}{\mathrm{d}l_1} = \frac{1}{2l_1} \int_{v_0 - tl_1(1-l_1) - \left(v_0 - \frac{1}{4}t\right)}^{v_0 - tl_1(3-l_1)} \left(-l_1\mu - \Delta - 2l_1 t\left(1 - 2l_1\right)\right) \mathrm{dG}\left(v_1\right)
$$

$$
+ \frac{1}{18tl_1} \int_{v_0 - tl_1(3-l_1)}^{v_0 + tl_1(3+l_1)} \left(3t\left(1 + l_1\right) - \mu\right)\left(tl_1\left(3 + l_1\right) - \Delta\right) \mathrm{dG}\left(v_1\right).
$$

Notice that at the upper limit $v_1 = v_0 - tl_1\left(3 - l_1\right)$ the first integrand takes the value $l_1\left(t\left(1 + 3l_1\right) - \mu\right) \geq 0$, where the inequality follows from $\mu \in [0, 1]$. Since the first integrand is decreasing in v_1, this implies that the first integral is positive. To see that the second integral is also positive, note first that the first factor of the integrand is positive (since $\mu \in [0, 1]$). The second factor is decreasing in v_1 and is equal to zero when v_1 is at the upper limit $v_1 = v_0 + tl_1\left(3 + l_1\right)$. This implies that the second integral is also positive which, in turn, proves (15). ∎